Feeder Reconfiguration in Distribution Networks based on Convex Relaxation of OPF

Qiuyu Peng, Yujie Tang and Steven H. Low

***Abstract*—The feeder reconfiguration problem chooses the on/off status of the switches in a distribution network in order to minimize a certain cost such as power loss. It is a mixed integer nonlinear program and hence hard to solve. In this paper we propose a heuristic algorithm that is based on the recently developed convex relaxation of the AC optimal power flow problem. The algorithm is computationally efficient and scales linearly with the number of redundant lines. It requires neither parameter tuning nor initialization for different networks. It successfully computes an optimal configuration on all four networks we have tested. Moreover we have proved that the algorithm solves the feeder reconfiguration problem optimally under certain conditions for the case where only a single redundant line needs to be opened. We also propose a more computationally efficient algorithm and show that it incurs a loss in optimality of less than 3% on the four test networks.**

***Index Terms*—Power Distribution, Nonlinear systems, Power system control, Feeder reconfiguration**

I. Introduction

A Primary distribution system consists of buses, distribution lines, and (sectionalizing and tie) switches that can be opened or closed. There are two types of buses. *Substation buses* (or just *substations*) are connected to a transmission network from which they receive bulk power, and *load buses*[1] that receive power from the substation buses. During normal operation the switches are configured so that

1) There is no loop in the network.
2) Each load bus is connected to a single substation.

Hence, there is a tree component rooted at each substation and we refer to each such component as a *feeder*. The optimal feeder reconfiguration (OFR) problem seeks to alter the on/off status of these switches, for the purpose of load balancing or loss minimization subject to the above two requirements, e.g., [2]–[5]. See also a survey in [6] for many early papers and references to some recent work in [7].

The OFR problem is a combinatorial (on/off status of switches) optimization problem with nonlinear constraints (power flow equations) and can generally be NP-hard. Various algorithms have been developed to solve the OFR problems. Following the convention in [7], they roughly fall into two categories: formal methods and heuristic methods.

Formal Methods: Formal methods solve the OFR problem using existing optimization approach. They usually require significant amount of computation time. In [5], the problem is solved using a simulated annealing technique where the problem is formulated as a multi-objective mixed integer constrained optimization. In [8], ordinal optimization is proposed to reduce the computational burden through order comparison and goal softening. In [9], the problem is solved using generalized Benders decompositions. In [10], a mixed integer linear programming solver is applied to solve the problem after linearizaiton of the power flow equations. In [7], the problem is formulated as a mixed integer nonlinear program which is then solved as a mixed integer convex program through the second-order cone program (SOCP) relaxation.

Heuristic Methods: Heuristic methods exploit structural properties to solve the OFR problem. They are usually more computationally efficient than formal methods. In [3], an "iterative branch exchange approach" is applied to OFR. The network is initialized with a feasible topology. At each iteration, an opened switch is closed and a closed switch is opened to reduce the cost and maintain the radial structure. The algorithm stops once a local minimum is reached, i.e. for each currently opened switch, closing it and opening another switch will not further decrease the cost. See [4], [11] for further developments on this approach. This approach has the advantage that the intermediate configuration is always feasible, hence we can terminate the algorithm at any iteration to obtain a feasible solution. However, the performance is sensitive to the initial configuration and sometimes it takes too many iterations for the algorithm to terminate. A different heuristic approach, first proposed in [2] termed "successive branch reduction approach" in this paper, assumes all the switches are initially closed and they are sequentially opened based on a given criteria until a radial configuration is reached. This approach has two major advantages: 1) unlike the "iterative branch exchange approach", no initialization is required; and 2) the number of iterations are bounded by the number of redundant lines, which is usually small in practice. Some developments on this approach include relaxing the binary variable representing the status on the switch [12], [13] and generalization to unbalanced network based on a constant current model [14].

Optimal feeder reconfiguration is a mixed integer nonlinear optimization problem and therefore NP-hard in general. To overcome the first difficulty (mixed integer optimization), we propose a heuristic approach that only involves solving a small

A preliminary version has appeared in [1].

*This work was supported by ARPA-E grant DE-AR0000226, National Science Council of Taiwan, R.O.C. grant NSC 103-3113-P-008-001 and DTRA grant 11376437.

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[1]Distributed generations are viewed as loads with negative real power injections in this paper.

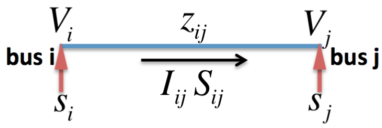


Fig. 1: Notations.

number of AC optimal power flow (OPF) problems and *no* mixed-integer optimization. We theoretically show that the proposed heuristic can obtain the global optimal solution under certain assumptions. Indeed global optimal configurations can always be found on the four practical networks in our simulations. To overcome the second difficulty (nonconvexity of AC OPF), we build on the recent development of SOCP relaxation of AC OPF. The effectiveness of this new approach is illustrated both through simulations of standard test systems and mathematical analysis under certain assumptions. Specifically the main contributions of the paper are twofold.

First, we propose an algorithm to optimize the "successive branch reduction approach". The algorithm uses a branch flow model introduced in [15], [16] for radial systems and exploit the recent development on solving the optimal power flow problem through convex relaxation [17]–[19]; see a tutorial in [20], [21] for more details. The algorithm has three major advantages:

1) Efficient: the complexity is linear in the number redundant lines that need to be opened.
2) Accurate: The algorithm is proved to solve OFR optimally under certain assumptions in the case where there is a single line that needs to be opened. Simulations on four practical networks show that it can find a globally optimal solution in the general case as well.
3) Hassle free: There are no parameters and initialization that need to be tuned for different networks.

Second, we simplify the above algorithm into one that has a constant complexity, i.e. the time complexity is independent of the number of redundant lines. Simulations on the same four practical networks show that the loss in optimality is less than 3%.

The rest of the paper is organized as follows. We formulate in Section II the optimal feeder reconfiguration problem. We propose and analyze in Section III our algorithms to solve the OFR problem when there is only one redundant line. The algorithms are extended in Section IV to general networks with arbitrary number of redundant lines. The simulation results are presented in Section V. We conclude in Section VI. All proofs are relegated to the Appendix.

II. Model and problem formulation

In this section, we define the optimal feeder reconfiguration (OFR) problem in a distribution network. We then review the optimal power flow (OPF) problem and how to solve it through the second-order cone programming (SOCP) relaxation.

A. Notations

We model a distribution network by a directed graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, where $\mathcal{N}$ represents the set of buses and $\mathcal{E}$ the set of lines connecting the buses in $\mathcal{N}$. We associate a direction with each line $(i,j)\in\mathcal{E}$ represented by an ordered pair of nodes in $\mathcal{N}$. There are two types of buses. Substation buses (or just substations) are connected to a transmission network from which they receive bulk power, and load buses that receive power from the substations. Let $\mathcal{N}_s$ denote the set of substation buses, $\mathcal{N}_l$ denote the set of load buses and $\mathcal{N}_s\cup\mathcal{N}_l=\mathcal{N}$.

For each bus $i\in\mathcal{N}$, let $V_i=|V_i|e^{\mathbf{i}\theta_i}$ be its complex voltage and $v_i:=|V_i|^2$ be its magnitude squared. Let $s_i=p_i+\mathbf{i}q_i$ be its net power injection which is defined as generation minus consumption. For each line $(i,j)\in\mathcal{E}$, let $z_{ij}=r_{ij}+\mathbf{i}x_{ij}$ be its complex impedance. Let I_{ij} be the complex branch current from buses i to j and $\ell_{ij}:=|I_{ij}|^2$ be its magnitude squared. Let $S_{ij}=P_{ij}+\mathbf{i}Q_{ij}$ be the branch power flow from buses i to j. For each line $(i,j)\in\mathcal{E}$, define S_{ji} in terms of S_{ij} and I_{ij} by $S_{ji}:=-S_{ij}+\ell_{ij}z_{ij}$. Hence $-S_{ji}$ represents the power received by bus j from bus i. The notations are illustrated in Fig. 1. A variable without a subscript denotes a column vector with appropriate components, as summarized below.

$s:=(s_i,i\in\mathcal{N})$	$v:=(v_i,i\in\mathcal{N})$
$S:=(S_{ij},(i,j)\in\mathcal{E})$	$\ell:=(\ell_{ij},(i,j)\in\mathcal{E})$

Given a graph $\mathcal{G}(\mathcal{N},\mathcal{E})$. For each node $i\in\mathcal{N}$, let

$$C(i):=\{(i,j)\mid(i,j)\in\mathcal{E}\}\cup\{(j,i)\mid(j,i)\in\mathcal{E}\},$$

which represents the set of lines with one end at i. For any $\mathcal{E}'\subseteq\mathcal{E}$, a path exists between two nodes $i,j\in\mathcal{N}$ in graph $\mathcal{G}(\mathcal{N},\mathcal{E}')$ if and only if there is a collection of edges in $\mathcal{E}'$ that connect node i and j. Denote

$$D^1_{\mathcal{E}'}:=\#\text{ of paths in }\mathcal{G}(\mathcal{N},\mathcal{E}')\text{ among buses in }\mathcal{N}_s \quad (1)$$

$$D^2_{\mathcal{E}'}:=\#\text{ of loops in }\mathcal{G}(\mathcal{N},\mathcal{E}') \quad (2)$$

$$D_{\mathcal{E}'}:=D^1_{\mathcal{E}'}+D^2_{\mathcal{E}'} \quad (3)$$

Given two real vectors $x,y\in\mathbb{R}^n$, $x\le y$ means $x_i\le y_i$ for $1\le i\le n$ and $x<y$ means $x_i<y_i$ for at least one component. The Pareto front (See [22] for more properties) of a compact set $A\subseteq\mathbb{R}^n$ is defined as

$$\mathcal{O}(A):=\{x\in A\mid\nexists\tilde{x}\in A\setminus\{x\}\text{ such that }\tilde{x}\le x\} \quad (4)$$

B. Problem formulation

There are sectionalizing or tie switches on the lines that can be opened or closed. Optimal feeder reconfiguration (OFR) is the problem of reconfiguring the switches to optimize certain objective subject to the topological constraints, power flow equations and operational constraints on voltage magnitudes and power injections. Typical objective includes optimizing total line loss, real power injection from the substations or load balancing. Let $\Gamma(s)$ denote the objective function. Then

- to minimize total line loss, we can set
$$\Gamma(s)=\sum_{i\in\mathcal{N}}\mathrm{Re}(s_i)=\sum_{i\in\mathcal{N}}p_i$$
- to minimize real power injection from the substations, we can set
$$\Gamma(s)=\sum_{i\in\mathcal{N}_s}\mathrm{Re}(s_i)=\sum_{i\in\mathcal{N}_s}p_i$$

- to balance loads for substations, we can set

$$\Gamma(s) = \sum_{i \in \mathcal{N}_s} |s_i|^2 = \sum_{i \in \mathcal{N}_s} (p_i^2 + q_i^2)$$

There are two topological constraints on configuring the switches during normal operations:

1) Each load bus is connected to a single substation.
2) There is no loop in the network.

Any subset of lines whose switches can be closed concurrently to satisfy both 1) and 2) is defined as a feasible configuration. Let $\mathcal{S}_T := \{\mathcal{E}_T \mid \mathcal{G}(\mathcal{N}, \mathcal{E}_T) \text{ satisfies 1) and 2)}\}$, which represents the set of all feasible configurations. When $|\mathcal{N}_s| = 1$, i.e. there is only one substation, $\mathcal{S}_T$ consists of the set of $\mathcal{E}_T$ such that $\mathcal{G}(\mathcal{N}, \mathcal{E}_T)$ is a spanning tree of $\mathcal{G}(\mathcal{N}, \mathcal{E})$.

We adopt the branch flow model first proposed in [15], [16] which has the phase angles of voltages and currents eliminated and uses only the variables $x := (s, S, \ell, v)$. For any $\mathcal{E}' \subseteq \mathcal{E}$, let $x(\mathcal{E}')$ represent the projection of x on graph $\mathcal{G}(\mathcal{N}, \mathcal{E}')$,i.e. $x(\mathcal{E}')$ collects all the variables in x except the branch power S_{ij} and branch current ℓ_{ij} for $(i,j) \in \mathcal{E} \setminus \mathcal{E}'$. The variables in $x(\mathcal{E}')$ satisfy:

$$s_i = -\sum_{(k,i)\in\mathcal{E}'} (S_{ki} - \ell_{ki} z_{ki}) + \sum_{(i,j)\in\mathcal{E}'} S_{ij}, \quad i \in \mathcal{N} \tag{5a}$$

$$v_j = v_i - 2\text{Re}(\overline{z}_{ij} S_{ij}) + \ell_{ij} |z_{ij}|^2, \ (i,j) \in \mathcal{E}' \tag{5b}$$

$$\ell_{ij} = \frac{|S_{ij}|^2}{v_i}, \quad (i,j) \in \mathcal{E}' \tag{5c}$$

Given a vector $x(\mathcal{E}')$ that satisfies (5), the phase angles of the voltages and currents can be uniquely determined if there is no loop in $\mathcal{G}(\mathcal{N}, \mathcal{E}')$. This is important for us since there is no loop in any feasible configurations $\mathcal{E}_T \in \mathcal{S}_T$, and therefore this relaxed model (5) is equivalent to the full AC power flow model; See [18, section III-A] for details.

In addition, there are also operational constraints on the power injection and voltage magnitude at each bus, i.e.

- *Power injection constraints:* for each bus $i \in \mathcal{N}$

$$s_i \in \{p + \mathbf{i}q \mid \underline{p}_i \le p \le \overline{p}_i, \underline{q}_i \le q \le \overline{q}_i\}. \tag{6a}$$

- *Voltage magnitude constraints:* for each bus $i \in \mathcal{N}$

$$\underline{v}_i \le v_i \le \overline{v}_i \tag{6b}$$

For instance, if the voltage magnitude at each bus is allowed to deviate by 5% from its nominal value, then $\underline{v}_i = 0.95^2$ and $\overline{v}_i = 1.05^2$.

For any configuration $\mathcal{E}' \subseteq \mathcal{E}$, let $\mathbb{X}(\mathcal{E}') := \{x(\mathcal{E}') \mid x(\mathcal{E}') \text{ satisfies (5) and (6)}\}$, which represents the feasible set of the $x(\mathcal{E}')$, then the OFR problem can be written as

$$\text{OFR}: \quad \min_{\mathcal{E}_T \in \mathcal{S}_T} \Gamma(s^*(\mathcal{E}_T)) \tag{7}$$

where

$$x^*(\mathcal{E}_T) := \arg\min_{x} \{\Gamma(s) \quad \text{s.t. } x(\mathcal{E}_T) \in \mathbb{X}(\mathcal{E}_T)\} \tag{8}$$

Different configurations $\mathcal{E}_T$ are implemented by different switch settings. OFR is difficult to solve due to the nonlinear feasible set $\mathbb{X}(\mathcal{E}_T)$ for a given configuration $\mathcal{E}_T$ and the discrete nature of $\mathcal{E}_T$. Before developing algorithms to solve OFR, we first review the optimal power flow (OPF) problem, on which our algorithms are based.

C. OPF and SOCP relaxation

The OPF problem seeks to optimize certain objective, e.g. total line loss or real power injection from the substations, subject to power flow equations (5) and operational constraints (6). Unlike the OFR problem, the OPF problem assumes a fixed switch configuration, i.e. it does not optimize over the topology of the network. For any $\mathcal{E}' \subseteq \mathcal{E}$ ($\mathcal{E}'$ is not required to be in $\mathcal{S}_T$), the OPF problem is:

$$\text{OPF-}\mathcal{E}': \quad \min_{x \in \mathbb{X}(\mathcal{E}')} \Gamma(s) \tag{9}$$

Note that the problem (8) is an instance of the OPF problem. The OPF problem (9) is noncovex due to the equalities in (5c). This is relaxed to inequalities in [17], [18]:

$$\ell_{ij} \ge \frac{|S_{ij}|^2}{v_i}, \quad (i,j) \in \mathcal{E}' \tag{10}$$

resulting in a (convex) second-order cone program (SOCP):

$$\text{ROPF-}\mathcal{E}': \quad \min_{x \in \mathbb{X}_c(\mathcal{E}')} \Gamma(s) \tag{11}$$

where

$$\mathbb{X}_c(\mathcal{E}') := \{x(\mathcal{E}') \mid x(\mathcal{E}') \text{ satisfies (5a), (5b), (10) and (6)}\}$$

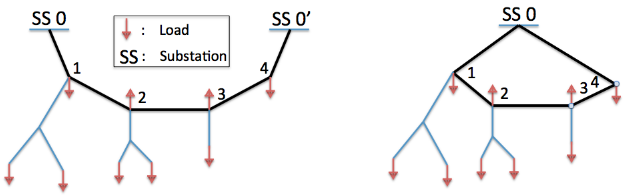
is the feasible set after relaxation. Clearly the relaxation ROPF (11) provides a lower bound for the original OPF problem (9) since the original feasible set $\mathbb{X}(\mathcal{E}') \subseteq \mathbb{X}_c(\mathcal{E}')$. The relaxation is called *exact* if every optimal solution of ROPF attains equalities in (5c) and hence is also optimal for the original OPF; see [20], [21] for more details. For a network with a tree topology, SOCP relaxation is exact under some mild conditions [18], [19] [2]. Throughout this paper, we assume the SOCP relaxation is always exact. Then we have the following result of [19, Theorem 3], which will be useful for us.

Theorem 1: Suppose ROPF is exact and the feasible set is nonempty. Then there exists a unique solution provided the objective function $\Gamma(s)$ is convex and nondecreasing in s.

III. Network with Single redundant line

In this section we consider the special case where there is only one redundant line that needs to be opened, *i.e.* $D_{\mathcal{E}} = 1$. We develop an algorithm to solve the OFR problem in this case and prove that the algorithm solves OFR optimally under certain assumptions. In addition, we simplify the above algorithm to reduce its computation complexity and incur negligible loss in optimality. We extend both algorithms to the general networks in the next section.

[2] In [23], SOCP relaxation is applied to a bus injection model. However, these two models are equivalent and SOCP relaxation is exact for the bus injection model if and only if it is exact for the branch flow model.



(a) $|\mathcal{N}_s| = 2$ and $|\mathcal{E}| = |\mathcal{N}| - 1$. (b) $|\mathcal{N}_s| = 1$ and $|\mathcal{E}| = |\mathcal{N}|$.

Fig. 2: Possible network topology with one redundant line.

A. Algorithms

When there is only one redundant line that needs to be opened, there are two possible cases as illustrated in Fig 2.

1) $|\mathcal{N}_s| = 2$ and $|\mathcal{E}| = |\mathcal{N}| - 1$, *i.e.* there are two substations and $|\mathcal{N}| - 1$ lines as shown in figure 2a. Then each load bus is connected to two substations and we need to open one line from the path between the two substations.
2) $|\mathcal{N}_s| = 1$ and $|\mathcal{E}| = |\mathcal{N}|$, *i.e.* there is one substation and $|\mathcal{E}| = |\mathcal{N}|$ lines as in figure 2b. Then there exists a loop and we need to open one line to break the loop.

The algorithm to solve both cases in Fig. 2 is stated in Algorithm 1. The basic idea of Algorithm 1 is simple and we illustrate it using the line network in Fig. 3. For the line network in Fig. 3, let the buses at the two ends be substation buses and buses in between be load buses. Then $\mathcal{N}_s := \{0, 0'\}$, $\mathcal{N}_l := \{1, \dots, n\}$ and $\mathcal{N} := \{0, 1, \dots, n, 0'\}$. We use $n+1$ and $0'$ interchangeably for notational convenience.

Algorithm 1 Network with one redundant line

$\mathcal{E}_T^* \leftarrow \mathcal{E}$
Solve OPF-$\mathcal{E}$ with an optima x^*
Calculate $\hat{e} \in \arg\min_e \{|P_e^*(\mathcal{E}_T^*)| \mid D_{\mathcal{E}_T^* \setminus e} = 0\}$
Denote $\hat{e} := (n_1, n_2)$
if $P_{\hat{e}} > 0$ **then**
  $e^* \leftarrow \arg\min_e \{\Gamma(p^*(\mathcal{E}_T^* \setminus e)) \mid e \in C(n_2)\}$
else
  $e^* \leftarrow \arg\min_e \{\Gamma(p^*(\mathcal{E}_T^* \setminus e)) \mid e \in C(n_1)\}$
end if
$\mathcal{E}_T^* \leftarrow \mathcal{E}_T^* \setminus e^*$
return $\mathcal{E}_T^*$

For the line network shown in Fig. 3, each load bus is connected to both substation 0 and $0'$, thus the set of feasible configuration is given as

$$\mathcal{S}_T := \{\mathcal{E} \setminus (k, k+1) \mid 1 \le k \le n\},$$

i.e. each line in $\mathcal{E}$ can be opened to create a feasible configuration. For each bus k, the set of lines with one end at it is given as

$$C(k) = \begin{cases} \{(k, k+1), (k-1, k)\} & k \ne 0, n+1 \\ \{(0, 1)\} & k = 0 \\ \{(n, n+1)\} & k = n+1 \end{cases}$$

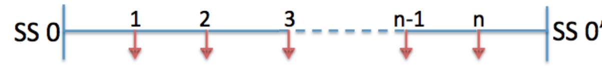


Fig. 3: A line Network

In Algorithm 1, we first solve OPF-$\mathcal{E}$, which provides an optimal solution x^* assuming all the lines are closed. Then we search for a branch $\hat{e}$, whose branch power flow is minimum in $\mathcal{E}$. Denote $\hat{e} = (k, k+1)$ and the line we will open is based on the following criteria:

1) $P_{\hat{e}} > 0$ and $k = n+1$: There is only one candidate, *i.e.* $C(n+1) = \{(n, n+1)\}$ and line $(n, n+1)$ is opened. It means substation $0'$ absorbs real power.
2) $P_{\hat{e}} > 0$ and $k < n+1$: There are two candidates, *i.e.* $C(k) = \{(k, k+1), (k+1, k+2)\}$. Either line $(k, k+1)$ or $(k+1, k+2)$ is opened, depending on which gives a smaller objective value.
3) $P_{\hat{e}} \le 0$ and $k = 0$: There is only one candidate, *i.e.* $C(0) = \{(0, 1)\}$ and line $(0, 1)$ is opened. It means substation 0 absorbs real power.
4) $P_{\hat{e}} \le 0$ and $k > 0$: There are two candidates, *i.e.* $C(k+1) = \{(k, k+1), (k-1, k)\}$. Either line $(k, k+1)$ or $(k-1, k)$ is opened, depending on which gives a smaller objective value.

The intuition behind Algorithm 1 is that the line which will be opened is close to the line where there is minimum branch flow power if we solve the problem assuming all the lines are closed (OPF-$\mathcal{E}$). Hence, we need to solve two other OPF problems for comparing the objective of the two candidates in addition to OPF-$\mathcal{E}$. Indeed, we can directly open the line with minimum branch power flow to simplify the algorithm after OPF-$\mathcal{E}$ is solved. By doing this, we sacrifice accuracy but simulation results show that the solution of the corresponding algorithm incurs a similar cost as that of Algorithm 1. The simplified algorithm is stated in Algorithm 2.

Algorithm 2 Network with one redundant line (simplified)

Solve OPF-$\mathcal{E}$ with an optima x^*.
Calculate $\hat{e} \in \arg\min_e \{|P_e^*| \mid D_{\mathcal{E} \setminus e} = 0\}$
$\mathcal{E}_T^* \leftarrow \mathcal{E} \setminus \hat{e}$
return $\mathcal{E}_T^*$

B. Performance analysis

We analyze the performance of Algorithm 1, *i.e.* whether the configuration $\mathcal{E}_T^*$ returned by Algorithm 1 is optimal for OFR. There are two possible cases as illustrated in Fig. 2. Case (b) can be reduced to case (a) by replacing the substation 0 by two virtual substations 0 and $0'$ as shown in Fig. 2a, where $\mathcal{N}_s := \{0, 0'\}$, $\mathcal{N}_l := \{1, \dots, n\}$. Hence, we only need to focus on case (a). For ease of presentation we only prove the results for a line network as shown in Fig. 3. They generalize in a straightforward manner to radial networks as shown in Fig. 2a. We make several assumptions below for our analysis:

A1 : $\overline{p}_i < 0$ for $i \in \mathcal{N}_l$ and $\overline{p}_i > 0$ for $i \in \mathcal{N}_s$.
A2 : $\overline{v}_i = \underline{v}_i = 1$ for $i \in \mathcal{N}$.

A3 : $|\theta_i - \theta_j| < \arctan(x_{ij}/r_{ij})$ for $(i,j) \in \mathcal{E}$.

A4 : The objective function $\Gamma(s) := \Gamma(p_0, p_{0'})$ is convex and increasing of $p_0, p_{0'}$.

A5 : The feasible set $\mathbb{X}(\mathcal{E})$ is compact.

A1 says that only substation buses 0 and $0'$ inject real power while load buses $1, \ldots, n$ absorb real power. A2 says that the voltage magnitude at each bus is fixed at their nominal value. A3 bounds the angle difference between adjacent buses.[3] A4 says that the objective function is merely a function of the power injections at two substations. A5 is a technical assumption that guarantees that our optimization problems are feasible.

The assumptions A1-A5 may not hold in practice, *e.g.* A1 is violated when there are distributed generators at some load buses, A2 is violated when buses have limited reactive power injection capability. However, we only need A1-A5 to make precise statements about the performance of Algorithm 1. We will first explain the intuition before formally stating the result in Theorem 2.

We now rewrite the OFR problem (7) for the line network in Fig. 3. Some new notations will be defined, which will only be used in this section. For any $(k, k+1) \in \mathcal{E}$, let $\mathcal{G}_0^k$ and $\mathcal{G}_{0'}^{k+1}$ represent the two subtrees rooted at 0 and $0'$ respectively if line $(k, k+1)$ is opened. Denote

$$(p_0^k, p_{0'}^{k+1}) := \left(p_0^*(\mathcal{E}_T^{k,k+1}), p_{0'}^*(\mathcal{E}_T^{k,k+1})\right), \tag{12}$$

where $\mathcal{E}_T^{k,k+1} := \mathcal{E} \setminus (k, k+1)$ and $x^*(\mathcal{E}_T^{k,k+1})$ is the optimal solution to a given configuration $\mathcal{E}_T^{k,k+1}$ and defined in (8). $(p_0^k, p_{0'}^{k+1})$ represents the minimum power injection at the substations for the two subtrees $\mathcal{G}_0^k$ and $\mathcal{G}_{0'}^{k+1}$ after line $(k, k+1)$ is opened. Then the OFR problem (7) for the line network (Fig. 3) can be written equivalently as

$$\min_{0 \le k \le n} \quad \Gamma(p_0^k, p_{0'}^{k+1}). \tag{13}$$

Define an OPF problem:

$$\text{OPF-}\mathcal{E}\text{s:} \qquad f(p_0) := \min_{x \in \mathbb{X}(\mathcal{E})} \quad p_{0'} \qquad \text{s.t.} \quad p_0 \text{ is a given constant} \tag{14}$$

Recall that $\mathbb{X}(\mathcal{E})$ is the feasible set of physical variables given a configuration $\mathcal{E}$ and $\mathbb{X}_c(\mathcal{E})$ is the convexified $\mathbb{X}(\mathcal{E})$. Let $\mathbb{P} := \{(p_0, p_{0'}) \mid \exists x \in \mathbb{X}(\mathcal{E})\}$ represent the projection of $\mathbb{X}(\mathcal{E})$ on $\mathbb{R}^2$ and $\mathbb{P}_c := \{(p_0, p_{0'}) \mid \exists x \in \mathbb{X}_c(\mathcal{E})\}$ be the projection of $\mathbb{X}_c(\mathcal{E})$ on $\mathbb{R}^2$. By definition of Pareto front in (4), the exactness of SOCP relaxation implies that $\mathcal{O}(\mathbb{P}) = \mathcal{O}(\mathbb{P}_c)$.

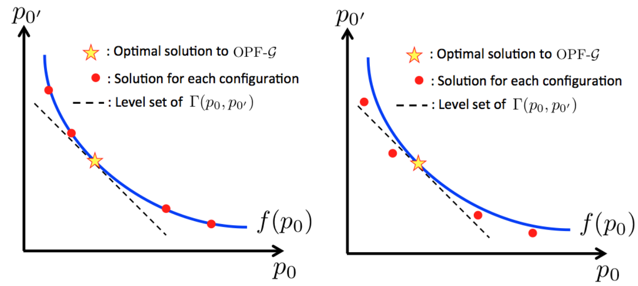
Lemma 1: Suppose A4-A5 hold and the SOCP relaxation is exact. Then

1) $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P}_c)$.
2) $f(p_0)$ is a strictly convex decreasing function of p_0.

By Lemma 1-1), $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P}_c)$, hence OPF-$\mathcal{E}$ can be written equivalently as

$$\min \quad \Gamma(p_0, f(p_0)), \tag{15}$$

[3]Although voltage phase angles θ_i are relaxed in the relaxed branch flow model (5), they are uniquely determined by $\theta_i - \theta_j = \angle(v_i - z_{ij}^* S_{ij})$ in a radial network [18].



(a) Intuition under A1-A5. (b) Intuition without A1-A5.

Fig. 4: Intuitions of Algoirthm 1.

In other words, solving OPF-$\mathcal{E}$ is equivalent to finding a point when the level set of $\Gamma(p_0, p_{0'})$ first hits the curve $(p_0, f(p_0))$ on a two dimensional plane, where the x-axis and y-axis are the real power injections from substation 0 and $0'$, as shown in Fig. 4. On the other hand, the OFR problem can be written as (13) and solving OFR is equivalent to find a point when the level set of $\Gamma(p_0, p_{0'})$ first hits one point in $\{(p_0^k, p_{0'}^{k+1}) \mid 0 \le k \le n\}$ on the two dimentional plane. Suppose A1-A5 hold, all the feasible points $(p_0^k, p_{0'}^{k+1})$ locate exactly on the curve $(p_0, f(p_0))$ as shown in Fig. 4a. Thus, we can obtain exactly the optimal solution to OFR by checking the points $(p_0^k, p_{0'}^{k+1})$ adjacent to the optimal solution to OPF-$\mathcal{E}$, which is performed in Algorithm 1. The result is formally stated in Theorem 2.

Theorem 2: Suppose A1–A5 hold. Then the configuration $\mathcal{E}_T^*$ returned by Algorithm 1 is optimal for OFR (7).

Remark: Theorem 2 shows that Algorithm 1 computes an optimal solution of OFR under assumptions A1-A5, which may not hold in practice. Without assuming A1-A5, $(p_0^k, p_{0'}^{k+1})$ does not locate exactly on the curve $(p_0, f(p_0))$ as shown in Fig. 4b. Thus, the points $(p_0^k, p_{0'}^{k+1})$ adjacent to the optimal solution to OPF-$\mathcal{E}$ may not be optimal for OFR. Indeed, we can create artificial examples to show that Algorithm 1 fails to find a global optimal configuration. However, the sub-optimality gap is usually small since the points $(p_0^k, p_{0'}^{k+1})$ are close to the the curve $(p_0, f(p_0))$. And global optimal configuration can always be found in our simulations on four practical networks.

IV. General Network Configuration

In section III, we propose two algorithms to solve the OFR problem assuming there is only one redundant line that needs to be open. In this section, we will extend both Algorithms 1 and 2 to general networks where there may be more than one redundant lines that need to be opened. As before, one of the algorithms has a higher accuracy but requires more computation (Algorithm 3) and the other lower accuracy but less computation (Algorithm 4).

Loosely speaking, Algorithm 1 consists of the following procedure:

1) Solve OPF problem assuming all the lines are closed.
2) Find the line $\hat{e}$ with minimum branch power flow.
3) Check line $\hat{e}$ against the lines adjacent to line $\hat{e}$ and the minimum of those lines as a solution.

For a general network, there are multiple lines that need to be simultaneously open. Then we generalize Algorithm 1 in the following manner: We iterate the procedure in Algorithm 1 and remove one line from $\mathcal{E}$ at the end of each iteration, resulting in a different OPF problem to solve for the next iteration. There are $|\mathcal{E}|-|\mathcal{N}_l|$ redundant lines and hence $|\mathcal{E}|-|\mathcal{N}_l|$ iterations. The algorithm is formally stated in Algorithm 3.

Algorithm 3 General Network

$\mathcal{E}_T^* \leftarrow \mathcal{E}$
while $D_{\mathcal{E}_T^*} > 0$ **do**
  Solve OPF-$\mathcal{E}_T^*$ with optima $x^*(\mathcal{E}_T^*)$
  Calculate $\hat{e} \in \arg\min_e \{|P_e^*(\mathcal{E}_T^*)| \mid D_{\mathcal{E}_T^* \setminus e} < D_{\mathcal{E}_T^*}\}$
  Denote $\hat{e} := (n_1, n_2)$
  if $P_{\hat{e}} > 0$ **then**
    $e^* \leftarrow \arg\min_e \{\Gamma(p^*(\mathcal{E}_T^* \setminus e)) \mid e \in C(n_2) \cap \mathcal{E}_T^*\}$
  else
    $e^* \leftarrow \arg\min_e \{\Gamma(p^*(\mathcal{E}_T^* \setminus e)) \mid e \in C(n_1) \cap \mathcal{E}_T^*\}$
  end if
  $\mathcal{E}_T^* \leftarrow \mathcal{E}_T^* \setminus e^*$
end while
return $\mathcal{E}_T^*$

Similarly, we can mimic Algorithm 2 and have an efficient algorithm which merely solve one OPF problem. Algorithm 2 consists of the following procedure:

1) Solve OPF problem assuming all the lines are closed.
2) Open the line $\hat{e}$ with minimum branch power flow.

We generalize Algorithm 2 in the following manner. We solve only one OPF problem OPF-$\mathcal{E}$, which assumes all the lines are closed. Then we sequentially choose one line with the smallest branch power flow in the remaining closed lines merely based on the solution to OPF-$\mathcal{E}$. Our simulations show that the simplification leads to negligible loss in optimality compared to Algorithm 3. The algorithm is stated in Algorithm 4.

Algorithm 4 General Network (simplified)

$\mathcal{E}_T^* \leftarrow \mathcal{E}$
Solve OPF-$\mathcal{E}$; let x^* be an optimal solution.
while $D_{\mathcal{E}_T^*} > 0$ **do**
  Calculate $\hat{e} \in \arg\min_e \{|P_e^*| \mid D_{\mathcal{E}_T^* \setminus e} < D_{\mathcal{E}_T^*}\}$
  $\mathcal{E}_T^* \leftarrow \mathcal{E}_T^* \setminus \hat{e}$
end while
return $\mathcal{E}_T^*$

Remark: Algorithm 3 scales linearly with the number of redundant lines and Algorithm 4 is independent of the number of redundant lines. For large distribution system, solving one OPF problem requires significant amount of time and Algorithm 4 can greatly reduce the computation time if there are many redundant lines.

V. Simulations

In this section we present examples to illustrate the effectiveness of the algorithms proposed in section IV (The algorithms in section III are special cases). We used a Macbook Pro with 2.9 Ghz Intel Core i7 and 8GB memory. The algorithms are implemented in Matlab 2013a and the OPF problem is solved using Gurobi optimization solver.

We test the algorithms on four practical distribution networks. Test network 1 is from Taiwan Power Company and the network data is taken from [24]. Test network 2 is from Brazil and the network data is taken from [25]. There are no renewable generations in these two networks. Test networks 3 and 4 are from Southern California Edition with renewable generations and taken from [17], [26]. Since the original data on these two networks consist of a single substation and contain no loop, we make several modifications to add loops in order to test our algorithms. The modified circuit diagram and network data of test network 3 are shown in Fig. 5 and Table I. The modified circuit diagram and network data of test network 4 are shown in Fig. 6 and Table II.

In the simulations, the voltage magnitude of the substations is fixed at 1 p.u. The voltage magnitudes at all other buses are allowed to vary within $[0.95, 1.05]$p.u. Our objective is to minimize the power loss, i.e. $\Gamma(s) := \sum_{i\in\mathcal{N}} p_i$. For all four networks, Algorithm 3 always computes an optimal configuration and Algorithm 4 computes a configuration with only up to 3% loss in optimality.

A. Case I: Tai-83 Bus System [24]

The Tai-83 bus system consists of 96 lines and 13 of them needs to be kept open to satisfy the configuration requirement. This network has been tested in [7], [24], [27]–[30] using different approaches. In [7], Jabr, *et. al* show that opening lines (7,13,34,39,42,55,62,72,83,86,89,90,92) gives an optimal solution using mixed integer convex programming solver. The results are summarized in Table III, where we also show the loss reduction[4], which represents the relative saving on power loss due to reconfiguration.

We run both Algorithm 3 and 4 for this network. Algorithm 3 returns the same optimal solution as [7], [29], [30]. However, Algorithm 3 is computationally very efficient since we only solved 39 OPF problems, which take 0.94 seconds on a laptop (MacBook). Algorithm 4 opens lines (7,13,33,39,42,63,72,82,84,86,89,90,92) with a power loss of 471.39KW. Compared with the optimal solution of 469.88KW, the difference in the power loss is less than 0.4% but we only need to solve 1 OPF problem, which takes 0.024 second on a laptop.

B. Case II: Brazil-135 Bus System [25]

The Brazil-135 bus system consists of 156 lines and 21 of them needs to be kept open to satisfy the configuration requirement. This network has been tested in [7], [25], [30] using different approaches. In [7], Jabr, *et. al* show that opening lines (7,35,51,90,96,106,118, 126,135,137,138,141,142, 144,145,146,147,148, 150,151,155) gives an optimal solution using mixed integer convex programming solver. The results are summarized in Table IV.

[4]loss reduction=$1-\frac{\text{power loss (after reconfiguration)}}{\text{power loss (before reconfiguration)}}$

TABLE I: Network of Fig. 5: Line impedances, peak spot load KVA, Capacitors and PV generation's nameplate ratings.

Network Data															
Line Data				Line Data				Line Data				Load Data		Load Data		PV Generators	
From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	Bus No.	Peak MVA	Bus No.	Peak MVAR	Bus No.	Nameplate Capacity
1	2	0.259	0.808	8	41	0.107	0.031	21	22	0.198	0.046	1	30	34	0.2		
2	13	0	0	8	35	0.076	0.015	22	23	0	0	11	0.67	36	0.27	13	1.5MW
2	3	0.031	0.092	8	9	0.031	0.031	27	31	0.046	0.015	12	0.45	38	0.45	17	0.4MW
3	4	0.046	0.092	9	10	0.015	0.015	27	28	0.107	0.031	14	0.89	39	1.34	19	1.5 MW
3	14	0.092	0.031	9	42	0.153	0.046	28	29	0.107	0.031	16	0.07	40	0.13	23	1 MW
3	15	0.214	0.046	10	11	0.107	0.076	29	30	0.061	0.015	18	0.67	41	0.67	24	2 MW
4	20	0.336	0.061	10	46	0.229	0.122	32	33	0.046	0.015	21	0.45	42	0.13		
4	5	0.107	0.183	11	47	0.031	0.015	33	34	0.031	0.015	22	2.23	44	0.45	Shunt Capacitors	
5	26	0.061	0.015	11	12	0.076	0.046	35	36	0.076	0.015	25	0.45	45	0.2	Bus	Nameplate
5	6	0.015	0.031	15	18	0.046	0.015	35	37	0.076	0.046	26	0.2	46	0.45	No.	Mvar
6	27	0.168	0.061	15	16	0.107	0.015	35	38	0.107	0.015	28	0.13				
6	7	0.031	0.046	16	17	0	0	42	43	0.061	0.015	29	0.13	V_{base}= 12.35kv		1	6
7	32	0.076	0.015	18	19	0	0	43	44	0.061	0.015	30	0.2	S_{base} = 1MW		3	1.2
7	8	0.015	0.015	20	21	0.122	0.092	43	45	0.061	0.015	31	0.07			37	1.8
8	40	0.046	0.015	20	25	0.214	0.046	1	12	0.076	0.146	32	0.13			47	1.8
8	39	0.244	0.046	21	24	0	0	1	30	0.116	0.146	33	0.27				

TABLE II: Network of Fig. 6: Line impedances, peak spot load KVA, Capacitors and PV generation's nameplate ratings.

Network Data																	
Line Data				Line Data				Line Data				Load Data		Load Data		Load Data	
From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	Bus No.	Peak MVA	Bus No.	Peak MVA	Bus No.	Peak MVA
1	2	0.160	0.388	20	21	0.251	0.096	39	40	2.349	0.964	3	0.057	29	0.044	52	0.315
2	3	0.824	0.315	21	22	1.818	0.695	34	41	0.115	0.278	5	0.121	31	0.053	54	0.061
2	4	0.144	0.349	20	23	0.225	0.542	41	42	0.159	0.384	6	0.049	32	0.223	55	0.055
4	5	1.026	0.421	23	24	0.127	0.028	42	43	0.934	0.383	7	0.053	33	0.123	56	0.130
4	6	0.741	0.466	23	25	0.284	0.687	42	44	0.506	0.163	8	0.047	34	0.067	Shunt Cap	
4	7	0.528	0.468	25	26	0.171	0.414	42	45	0.095	0.195	9	0.068	35	0.094	Bus	Mvar
7	8	0.358	0.314	26	27	0.414	0.386	42	46	1.915	0.769	10	0.048	36	0.097	19	0.6
8	9	2.032	0.798	27	28	0.210	0.196	41	47	0.157	0.379	11	0.067	37	0.281	21	0.6
8	10	0.502	0.441	28	29	0.395	0.369	47	48	1.641	0.670	12	0.094	38	0.117	30	0.6
10	11	0.372	0.327	29	30	0.248	0.232	47	49	0.081	0.196	14	0.057	39	0.131	53	0.6
11	12	1.431	0.999	30	31	0.279	0.260	49	50	1.727	0.709	16	0.053	40	0.030	Photovoltaic	
11	13	0.429	0.377	26	32	0.205	0.495	49	51	0.112	0.270	17	0.057	41	0.046	Bus	Capacity
13	14	0.671	0.257	32	33	0.263	0.073	51	52	0.674	0.275	18	0.112	42	0.054		
13	15	0.457	0.401	32	34	0.071	0.171	51	53	0.070	0.170	19	0.087	43	0.083	45	5MW
15	16	1.008	0.385	34	35	0.625	0.273	53	54	2.041	0.780	22	0.063	44	0.057		
15	17	0.153	0.134	34	36	0.510	0.209	53	55	0.813	0.334	24	0.135	46	0.134	V_{base} = 12kV	
17	18	0.971	0.722	36	37	2.018	0.829	53	56	0.141	0.340	25	0.100	47	0.045	S_{base} = 1MVA	
18	19	1.885	0.721	34	38	1.062	0.406	1	32	0.113	0.434	27	0.048	48	0.196	Z_{base} = 144Ω	
4	20	0.138	0.334	38	39	0.610	0.238	53	57	0.1	0.3	28	0.038	50	0.045		
19	58	0.09	0.2														

Algorithm 3 computes the same optimal solution as [7], [30]. However, Algorithm 3 is computationally very efficient since we only solved 63 OPF problems, which take 2.2 seconds on a laptop. Algorithm 4 opens lines (35,51,55, 84,90,106,126,135,136,137,138,141,143,144,145,147,148,152, 150,151,155) with a power loss of 288.01KW. Compared with the optimal solution of 280.19KW, the difference in the power loss is less than 2.8% but we only need to solve 1 OPF problem, which takes 0.055 second on a laptop.

C. Case III: SCE-47 Bus System

TABLE III: Summary on Tai-83 Bus System

Method	Opened Lines	Losses (KW)	Loss reduction
[24], [27], [28]	7,13,34,39,41,55,62, 72,83,86,89,90,92	471.08	11.45%
[7], [29], [30]	7,13,34,39,42,55,62, 72,83,86,89,90,92	469.88	11.68%
Algorithm 3	7,13,34,39,42,55,62, 72,83,86,89,90,92	469.88	11.68%
Algorithm 4	7,13,33,39,42,63,72, 82,84,86,89,90,92	471.39	11.40%

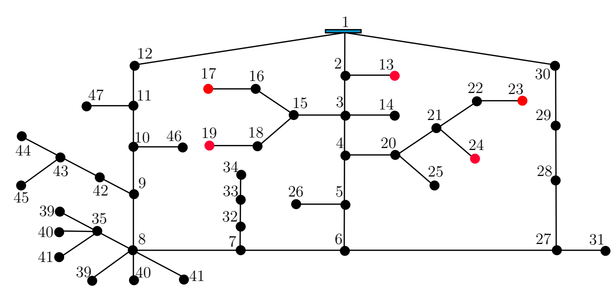


Fig. 5: A modified SCE 47-bus feeder. The blue bar (1) represents the substation bus, the red dots (13, 17, 19, 23, 24) represent buses with PV panels, and the other dots represent load buses without PV panels.

The original data for the SCE 47-bus system does not contain loops, so we added two lines to connect the substation bus 1 to two load buses 12 and 30 respectively. Hence there are 49 lines and 2 of them needs to be open in the modified feeder. In addition to the loads, there are 5 PV panels and their power injections can be controlled. The nameplates for these 5 PV panels can be found in Table I.

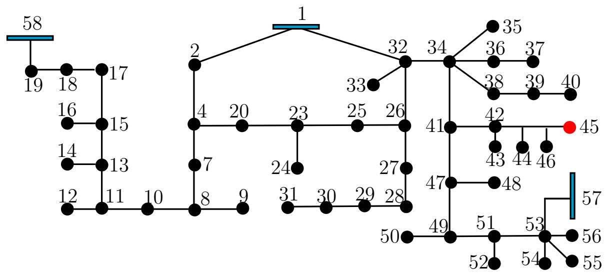


Fig. 6: A modified SCE 56-bus feeder. The blue bars (1, 57, 58) represent the substation buses and the red dot (45) represents the bus with PV panels.

There are in total 95 feasible configurations. We first calculate the objective value of all the 95 configurations. The best configuration is opening lines $\{(1, 2), (8, 9)\}$, resulting in 32.6KW power loss. The average power loss is 63.7KW and the worst configuration's power loss is 136.9KW across the 95 configurations. Hence the average power loss is almost twice as bad as the minimum power loss and the worst configuration is 4 times as bad as the minimum!

TABLE IV: Summary on Brazil-135 Bus System

Method	Opened Lines	Losses (KW)	Loss reduction
[25]	51,106,136,137,138,139, 141,142,143,144,145, 146,147,148,149,150, 151,152,154,155,156	285.77	10.80%
[7], [30]	7,35,51,90,96,106,118, 126,135,137,138,141,142, 144,145,146,147,148, 150,151,155	280.19	12.54%
Algorithm 3	7,35,51,90,96,106,118, 126,135,137,138,141,142, 144,145,146,147,148, 150,151,155	280.19	12.54%
Algorithm 4	35,51,55,84,90,106,126, 135,136,137,138,141,143, 144,145,147,148,152,150, 151,155	288.01	10.10%

Both Algorithms 3 and 4 find the optimal configuration for this network. Algorithm 3 solves 4 OPF problems (0.055 second) and Algorithm 4 solves 1 OPF problem (0.014 second). Compared with solving one OPF problem for each configuration to obtain the optimal solution, both algorithms are much more efficient without any loss in optimality.

D. Case IV: SCE-56 Bus System

In contrast to the SCE-47 system, where there are 5 relatively small PV panels, the SCE 56-bus system consists of a single big PV system with a capacity of 5MW. We make the following modifications:

- We add a line between bus 1 and bus 32 to create a loop.
- We assume there are two additional substations (bus 57 and 58): attached to substation 19 and 53, respectively.

There are 59 lines and 3 lines need to be kept open. There are in total 724 feasible configurations. We first calculate the objective value of all the 724 configurations. The best configuration is opening lines $\{(11, 13), (23, 25), (41, 47)\}$, resulting in 9.89KW power loss. The average power loss is 23.4KW and the worst power loss is 211KW across the 724 configurations.

We run both Algorithm 3 and 4 for this network. Algorithm 3 computes the optimal solution by opening lines $\{(11, 13), (23, 25), (41, 47)\}$ but solves just 9 OPF problems, which take 0.14 seconds. Algorithm 4 opens lines $\{(11, 13), (23, 25), (47, 49)\}$ with a power loss of 9.92KW. Compared with the optimal solution of 9.89KW, the difference in the power loss is less than 0.3% but Algorithm 4 only needs to solve one OPF problem, which takes 0.015 seconds.

VI. Conclusion

We have proposed two algorithms with different tradeoffs on efficiency and accuracy for feeder reconfiguration, based on the SOCP relaxation of OPF. We have proved that the algorithm solves OFR optimally under certain assumptions for a special case where there is only a single redundant line that needs to be opened, and have shown that the gap is very small. We have also demonstrated the effectiveness of our algorithms through simulations on four practical networks.

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APPENDIX A
PROOF OF LEMMA 1

We will first show that $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P}_c)$. Since $\mathcal{O}(\mathbb{P}_c) = \mathcal{O}(\mathbb{P})$, it is equivalent to prove $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P})$.

By property of *Pareto Front* [22], for each point $(p_0, p_{0'}) \in \mathcal{O}(\mathbb{P}_c) = \mathcal{O}(\mathbb{P})$, there exists a convex nondecreasing function $\Gamma^* : \mathbb{R}^2 \to \mathbb{R}$ such that $(p_0, p_{0'})$ is an optima for OPF-$\mathcal{E}$. Given any $(p_0, \hat{p}_{0'}) \in \mathcal{O}(\mathbb{P})$, let $\Gamma^*(p_0, p_{0'})$ be the objective function such that $(p_0, \hat{p}_{0'})$ solves OPF-$\mathcal{E}$. Since $\Gamma^*(p_0, p_{0'})$ is a nondecreasing function, OPF-$\mathcal{E}$ can be written equivalently as

$$\min_{p_0} \Gamma^*(p_0, p_{0'}) \quad \text{s.t. } p_{0'} = f(p_0)$$

Therefore, at optimality, $f(p_0) = \hat{p}_{0'}$ and $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P})$.

Next, we prove $f(p_0)$ is a strictly convex decreasing function of p_0.

Lemma 2: Let A be a compact and convex set in $\mathbb{R}^2$. Define $g(x) := y$ for any $(x, y) \in \mathcal{O}(A)$. Then $y = g(x)$ is a convex decreasing function of x for $(x, y) \in \mathcal{O}(A)$.

Proof: We first show $g(x)$ is a decreasing function and then show $g(x)$ is also convex.

Let $(x_1, g(x_1))$ and $(x_2, g(x_2))$ be two points in $\mathcal{O}(A)$. Without loss of generality, assume $x_1 > x_2$. If $g(x_1) \geq g(x_2)$, it violates the fact that $(x_1, g(x_1)) \in \mathcal{O}(A)$ and hence $g(x_1) < g(x_2)$, which means that $g(x)$ is a decreasing function.

Next, we will show $g(\cdot)$ is convex. Recall that A is a compact set, we have $(x_1, g(x_1)), (x_2, g(x_2)) \in \mathcal{O}(A) \subseteq A$. A is also a convex set, thus $(\frac{x_1+x_2}{2}, \frac{g(x_1)+g(x_2)}{2}) \in A$. By definition of Pareto front,

$$g(\frac{x_1 + x_2}{2}) = \inf_{(\frac{x_1+x_2}{2}, y) \in A} \{y\} \leq \frac{g(x_1) + g(x_2)}{2},$$

which shows $g(x)$ is a convex function. ■

Since $\mathbb{X}_c(\mathcal{G})$ is convex and compact by A5, its projection on a two dimensional space $\mathbb{P}_c$ is also compact and convex. Note that $(p_0, f(p_0)) \in \mathcal{O}(\mathbb{P}_c)$ by part 1) of Lemma 1, we have part 2).

APPENDIX B
PROOF OF THEOREM 2

For the line network in Fig. 3, denote $\hat{e} = (\hat{k}, \hat{k}+1)$ (Line 4 in Algorithm 1). Without loss of generality, assume $P_{\hat{k},\hat{k}+1} > 0$ and we need to show that e^* is the optimal line to open for OFR, i.e. either $(\hat{k}, \hat{k}+1)$ or $(\hat{k}+1, \hat{k}+2)$ will be opened for the optimal solution.

Based on Theorem 1, there exists a unique solution x^* for any OPF problems with convex nondecreasing objective function. Hence there is also a unique solution x^* to OPF-$\mathcal{E}s$ for any feasible real power injection p_0 at substation 0. In other words, x^* is a function of p_0 and let $x(p_0) := (s^*, S^*)$ represents the solution to OPF-$\mathcal{E}s$ with real power injection p_0 at substation 0. We skip v and ℓ in x since v_i is fixed by A2 and $\ell_{k,k+1}$ is uniquely determined by $S_{k,k+1}$ according to (5c). By Maximum theorem, $x(p_0)$ is a continuous function of p_0.

Lemma 3: Suppose A2-A5 hold. Then $P_{k,k+1}(p_0)$ is an increasing function of p_0 for all $(k, k+1) \in \mathcal{E}$.

Proof: See Appendix C for the proof. ■

Since $P_{k,k+1}(p_0)$ is an increasing and continuous function of p_0, there exists a unique p_0 to $P_{k,k+1}(p_0) = 0$ and denote $p_0(k) := P_{k,k+1}^{-1}(0)$, i.e. $P_{k,k+1}(p_0(k)) = 0$.

Lemma 4: Suppose A1-A5 hold. Then $p_0(k) < p_0(k+1)$ for $0 \leq k \leq n$.

Proof: By A1 ($\overline{p}_i < 0$ for $i \in \mathcal{N}_l$) and (5a), we have for $0 \leq k < n$,

$$P_{k,k+1}(p_0) = P_{k+1,k+2}(p_0) - p_{k+1} > P_{k+1,k+2}(p_0).$$

which means

$$0 = P_{k,k+1}(p_0(k)) > P_{k+1,k+2}(p_0(k)).$$

By Lemma 3, $P_{k+1,k+2}(p_0)$ is a increasing function of p_0, hence $p_0(k) < p_0(k+1)$. ■

Recall that $(p_0^k, p_{0'}^{k+1})$ is the minimal real power injection for subtrees $\mathcal{G}_0^k$ and $\mathcal{G}_{0'}^{k+1}$ respectively. Our next result shows that $(p_0^k, p_{0'}^{k+1}) = (p_0(k), f(p_0(k)))$.

Lemma 5: Suppose A2-A5 hold and the voltage magnitude of all buses are fixed at the same value. Then

$$(p_0^k, p_{0'}^{k+1}) = (p_0(k), f(p_0(k))) \quad (k, k+1) \in \mathcal{E}$$

Proof: By definition, $P_{k,k+1}(p_0(k)$. Based on (5b)-(5c),

$$2x_{k,k+1}Q_{k,k+1}(p_0(k)) = \ell_{k,k+1}(p_0(k))|z_{k,k+1}|^2$$
$$\ell_{k,k+1}(p_0(k)) = \frac{Q^2_{k,k+1}(p_0(k))}{v_k}$$

if $v_k = v_{k+1}$. Solving the above two equations gives $Q_{k,k+1}(p_0(k)) = 0$. Hence, $p_0(k)$ is a feasible power injection for subtree $\mathcal{G}_0^k$ and it means $p_0^k \leq p_0(k)$. Next, we will show that $p_0(k)$ is the smallest possible power injection for $\mathcal{G}_0^k$. Suppose we have $p_0^k < p_0(k)$, then $(p_0^k, f(p_0(k)))$ is a feasible power injection for network $\mathcal{G}$ with $p_0^k < p_0(k)$. It contradicts $(p_0(k), f(p_0(k))) \in \mathcal{O}(\mathbb{P})$ (Lemma 1). Therefore we have $p_0^k = p_0(k)$ and $p_{0'}^{k+1} = f(p_0(k))$. ■

By Lemma 1, $(p_0(k), f(p_0(k))) \in \mathcal{O}(\mathbb{P})$. Then Lemma 5 means the minimal power injection for each partition of graph $\mathcal{G}$ locates exactly on the Pareto front of the feasible power injection region of OPF-$\mathcal{E}$. Therefore the OFR problem (13) is equivalent to the following problem:

$$\min_{0\leq k\leq n} \Gamma(p_0^k, p_{0'}^{k+1}) = \min_{0\leq k\leq n} \Gamma(p_0(k), f(p_0(k))), \qquad (16)$$

whose minimizer is denoted by k^*.

On the other hand, by (15), OPF-$\mathcal{E}$ can be rewritten as

$$\min_{p_0 \in I_{p_0}} \Gamma(p_0, f(p_0)),$$

whose unique minimizer is denoted by p_0^*.

Lemma 6: Suppose A1-A5 and $P_{\hat{k},\hat{k}+1}(p_0^*) > 0$ hold. Then,

$$p_0(\hat{k}) \leq p_0^* \leq p_0(\hat{k}+1)$$

Proof: By our assumption at the beginning of the proof, $P_{\hat{k},\hat{k}+1}(p_0^*) > 0$, which implies that $p_0(\hat{k}) \leq p_0^*$. since $P_{\hat{k},\hat{k}+1}(p_0)$ is a increasing function of p_0 based on Lemma 3. On the other hand, by A1

$$P_{\hat{k},\hat{k}+1}(p_0^*) = P_{\hat{k}+1,\hat{k}+2}(p_0^*) - p_{\hat{k}+1}(p_0^*) > P_{\hat{k}+1,\hat{k}+2}(p_0^*),$$

implying that $P_{\hat{k}+1,\hat{k}+2}(p_0^*) < 0$. Otherwise $|P_{\hat{k},\hat{k}+1}(p_0^*)| > |P_{\hat{k}+1,\hat{k}+2}(p_0^*)|$, contradicting that $(\hat{k}, \hat{k}+1) = \arg\min_e\{|P_e^*(\mathcal{E})|\}$. Therefore, $p_0^* \leq p_0(\hat{k}+1)$ since $P_{\hat{k}+1,\hat{k}+2}(p_0)$ is an increasing function of p_0 by Lemma 3. ■

Considering Lemma 4 and 6, we have

$$p_0(k) \leq p_0^* \quad k \leq \hat{k}$$
$$p_0(k) > p_0^* \quad k > \hat{k}$$

Since $\Gamma(p_0, f(p_0))$ is a convex function with minimizer p_0^*, we have for $k_1 \leq k_2 \leq \hat{k}$

$$\Gamma(p_0(k_1), f(p_0(k_1))) \geq \Gamma(p_0(k_2), f(p_0(k_2))) \geq \Gamma(p_0^*, f(p_0^*)).$$

For $k_1 \geq k_2 \geq \hat{k}$,

$$\Gamma(p_0(k_1), f(p_0(k_1))) \leq \Gamma(p_0(k_2), f(p_0(k_2))) \leq \Gamma(p_0^*, f(p_0^*))$$

which indicates the OFR problem (16) can be reduced to

$$\min_{0\leq k\leq n} \Gamma(p_0(k), f(p_0(k))) = \min_{k=\hat{k},\hat{k}+1} \Gamma(p_0(k), f(p_0(k))),$$

which is solved in line 6 of Algorithm 1. Hence Algorithm 1 solves the optimal solution to OFR.

APPENDIX C
PROOF OF LEMMA 3

For a line $(k, k+1)$ between two buses k and $k+1$ with fixed voltage magnitude, $(S_{k,k+1}, \ell_{k,k+1})$ are governed by (5b)-(5c) and $Q_{k,k+1}, \ell_{k,k+1}$ can be united solved given a $P_{k,k+1}$ if A3 holds. Denote $\phi(P_{k,k+1}) := -P_{k,k+1} = P_{k,k+1} - \ell_{k,k+1}r_{k,k+1}$ and we have the following result.

Lemma 7: Suppose A2 and A3 hold, $\phi(P_{k,k+1})$ is a concave increasing function of $P_{k,k+1}$ for $(k, k+1) \in \mathcal{E}$.

Proof: By (5c), we have $\ell_{k,k+1} = (P^2_{k,k+1} + Q^2_{k,k+1})/v_i$ and substitute it in $\phi(P_{k,k+1})$, we have

$$\phi(P_{k,k+1}) = P_{k,k+1} - \frac{r_{k,k+1}}{v_i}\left(P^2_{k,k+1} + Q^2_{k,k+1}\right).$$

The relation between $P_{k,k+1}$ and $Q_{k,k+1}$ is governed by (5b). Let $\theta_{k,k+1} := \theta_i - \theta_j$ and then $P_{k,k+1}$ and $Q_{k,k+1}$ can be written as

$$P_{k,k+1} = \frac{v_i r_{k,k+1}}{|z_{k,k+1}|^2} + \sqrt{\frac{v_i v_j}{|z_{k,k+1}|^2}}\sin(\theta_{k,k+1} - \beta_{k,k+1})$$
$$Q_{k,k+1} = \frac{v_i x_{k,k+1}}{|z_{k,k+1}|^2} - \sqrt{\frac{v_i v_j}{|z_{k,k+1}|^2}}\cos(\theta_{k,k+1} - \beta_{k,k+1}),$$

where $\beta_{k,k+1} := \arctan r_{k,k+1}/x_{k,k+1}$. Substitute them into $\phi(P_{k,k+1})$, we obtain

$$\phi(P_{k,k+1}) = -\frac{v_j r_{k,k+1}}{|z_{k,k+1}|^2} + \sqrt{\frac{v_i v_j}{|z_{k,k+1}|^2}}\sin(\theta_{k,k+1} + \beta_{k,k+1}).$$

Take derivative of $\phi(P_{k,k+1})$ with respect to $P_{k,k+1}$, we have

$$\frac{d\phi(P_{k,k+1})}{dP_{k,k+1}} = \frac{\cos(\theta_{k,k+1} + \beta_{k,k+1})}{\cos(\theta_{k,k+1} - \beta_{k,k+1})}.$$

which is always positive by assumption A3 that $|\theta_{k,k+1}| < \arctan x_{k,k+1}/r_{k,k+1}$. Furthermore,

$$\frac{d^2\phi(P_{k,k+1})}{dP^2_{k,k+1}} = -\sqrt{\frac{|z_{k,k+1}|^2}{v_i v_j}}\frac{\sin 2\beta_{k,k+1}}{\cos^3(\theta_{k,k+1} - \beta_{k,k+1})},$$

which is always negative by assumption A3 that $|\theta_{k,k+1}| < \arctan x_{k,k+1}/r_{k,k+1}$. Thus, $\phi(P_{k,k+1})$ is a concave increasing function of $P_{k,k+1}$. ■

Lemma 7 means if the one end of the line increases its real power injection on the line, the other end should receive more real power under assumption A2 and A3. We now show that $P_{k,k+1}(p_0)$ is a nondecreasing function of p_0 for all $(k, k+1) \in \mathcal{E}$.

Suppose $P_{k,k+1}(p_0)$ is not a nondecreasing function of p_0 at p_0^* for a line $(k, k+1) \in \mathcal{E}$, then either C1 or C2 below will hold for arbitrary small $\epsilon > 0$,

C1: $\exists p_0 \in (p_0, p_0^* + \epsilon)$ such that $P_{k,k+1}(p_0) < P_{k,k+1}(p_0^*)$.
C2: $\exists p_0 \in (p_0 - \epsilon, p_0^*)$ such that $P_{k,k+1}(p_0) > P_{k,k+1}(p_0^*)$.

We will show by contradiction that $(p_0^*, f(p_0^*)) \notin \mathcal{O}(\mathbb{P})$ in this case, which violates Lemma 1. Assume without loss of generality that $P_{i,i+1}(p_0)$ is a nondecreasing function of p_0 for $0 \le i < k$.

Case I: $q_k(p_0^*) > \underline{q}_k$. Suppose C1 holds, then there exists a monotone decreasing sequence $p_0^{(m)} \downarrow p_0^*$ such that $\{P_{k,k+1}(p_0^{(m)}), m \in \mathbb{N}\}$ is a monotone increasing sequence that converges to $P_{k,k+1}(p_0^*)$ because $x(p_0)$ is continuous over p_0. By power balance equation (5b) at bus k, for any m, we have

$$\begin{aligned} p_k(p_0^{(m)}; \mathcal{G}) &= P_{k,k+1}(p_0^{(m)}) - \phi(P_{k-1,k}(p_0^{(m)})) \\ &< P_{k,k+1}(p_0^{(m+1)}) - \phi(P_{k-1,k}(p_0^{(m+1)})) \\ &= p_k(p_0^{(m+1)}) \end{aligned}$$

Thus $\{p_k(p_0^{(m)}), n \in \mathbb{N}\}$ is a monotone increasing sequence that converges to $p_k(p_0^*)$. We now construct a point $\tilde{x} = (\tilde{P}, \tilde{Q}, \tilde{p}, \tilde{q})$ as follows. First, pick up $(\tilde{P}_{k,k+1}, \tilde{Q}_{k,k+1}, \tilde{p}_k, \tilde{q}_k)$ such that $\tilde{p}_k \in \{p_k(p_0^{(m)}; \mathcal{G}), m \in \mathbb{N}\}$, $\tilde{q}_k \in (\underline{q}_k, q_k(p_0^*))$ and they satisfy the following equations:

$$\tilde{P}_{k,k+1} = P_{k,k+1}(p_0^*) - p_k(p_0^*) + \tilde{p}_k \tag{17a}$$
$$\tilde{Q}_{k,k+1} = Q_{k,k+1}(p_0^*) - q_k(p_0^*) + \tilde{q}_k \tag{17b}$$
$$v_{k+1} = v_k - 2(r_{k,k+1}\tilde{P}_{k,k+1} + x_{k,k+1}\tilde{Q}_{k,k+1}) \tag{17c}$$
$$+ \frac{\tilde{P}_{k,k+1}^2 + \tilde{Q}_{k,k+1}^2}{v_k} |z_{k,k+1}|^2 \tag{17d}$$

The existence of $(\tilde{P}_{k,k+1}, \tilde{Q}_{k,k+1}, \tilde{p}_k, \tilde{q}_k)$ is guaranteed by the following two facts:

- $p_k(p_0^{(m)})$ is a monotone increasing sequence that converges to $p_k(p_0^*)$.
- $\tilde{q}_k$ is a continuous decreasing function of $\tilde{p}_k$ if they satisfy (17).

Since $\tilde{P}_{k,k+1} \in [P_{k,k+1}(p_0^{(1)}), P_{k,k+1}(p_0^*)]$ and $x(p_0)$ are continuous over p_0, then there exists a $p_0' \in [p_0^*, p_0^{(1)}]$ such that $S_{k,k+1}(p_0') = \tilde{S}_{k,k+1}$.

Next, we will construct the feasible physical variable for $i \neq k$. For $0 \le i < k$, let $\tilde{s}_i = s_i(p_0^*)$ and $\tilde{S}_{i,i+1} = S_{i,i+1}(p_0^*)$. For $k < i \le n$, let $\tilde{s}_i = s_i(p_0')$ and $\tilde{S}_{i,i+1} = S_{i,i+1}(p_0')$. Clearly that $\tilde{x} \in \mathbb{X}(\mathcal{E})$ with $(p_0^*, f(p_0'))$ as the real power injection at substation 0 and $0'$. However, $f(p_0') < f(p_0^*)$, which contradicts $(p_0^*, f(p_0^*)) \in \mathcal{O}(\mathbb{P})$.

Case II: $q_k(p_0^*) < \overline{q}_k$. Similar approach can be used to show C2 does not hold by contradiction.

So far, we have shown that $P_{k,k+1}(p_0)$ is non-decreasing either on its left or right neighborhood. Thus $P_{k,k+1}(p_0)$ is non-decreasing of p_0 if $\underline{q}_k < \overline{q}_k$ because $P(p_0)$ is a continuous function of p_0. The case where $\underline{q}_k = \overline{q}_k$ can be covered by taking limitation of the case of $\underline{q}_k < \overline{q}_k$.

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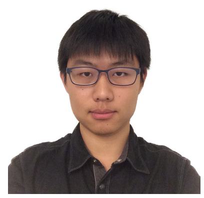

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