Optimal Investment of Conventional and Renewable Generation Assets

Desmond W.H. Cai
Department of Electrical Engineering
California Institute of Technology
Pasadena, California 91125
Email: wccai@caltech.edu

Yunjian Xu
Center for Mathematics and Information
California Institute of Technology
Pasadena, California 91125
Email: yunjian@caltech.edu

Steven H. Low
Department of Computer Science
California Institute of Technology
Pasadena, California 91125
Email: slow@caltech.edu

***Abstract*—Driven by the national policy to expand renewable generation, as well as the advances in renewable technologies that reduce the cost of small-scale renewable generation units, distributed generation at end users will comprise a significant fraction of electricity generation in the future. We study the problem faced by a social planner who seeks to minimize the long-term discounted costs (associated with both the procurement and the usage of conventional and distributed generation assets), subject to meeting an inelastic demand for electricity. Under mild conditions on the problem parameters, we fully characterize the optimal investment policy for the social planner. We also analyze the impact of problem parameters (e.g., asset lifespans) on the optimal investment policy through numerical examples.**

I. Introduction

Renewable generation capacity is expanding rapidly to potentially reduce carbon dioxide emissions and dependence on fossil fuels. The advances in renewable technologies has driven down the prices of renewable generation assets (e.g., solar photovoltaic (PV) devices), and makes it economically feasible to support a significant fraction of system load by renewable generation. According to the US Department of Energy, 20% of the US electricity capacity should come from renewable sources [1] by 2030, while the current levels of renewable penetration is less than 5%.

Driven by the national policy to expand renewable generation, as well as the cost reduction for small-scale renewable generation units, the future power system is expected to include a significant fraction of generation at end-users. As a result, investment strategies on distributed renewable generation have received much recent attention [2], [3], [4]. Most of existing works in this literature explore the investment problem through empirical (or numerical) approaches. The objective of this work, however, is to provide characterizations of the optimal investment policy.

In this paper, we study the tradeoff between investments in two types of assets, which have different investment prices, depreciation rates, and variable costs (for deployment). A social planner seeks to minimize the long-term discounted costs (associated with both the procurement and the usage of two types of generation assets), subject to meeting an inelastic demand for electricity. We note that this is a nontrivial sequential decision making problem, as the planner has to take into account a variety of factors including the depreciation of existing capacities, future demands for electricity, as well as future investment and variable costs. Under mild conditions on the problem parameters, we characterize the optimal investment policy for the social planner. In particular, we provide closed-form expressions for the time period at which investment switches from conventional to distributed (renewable) generation assets. We analyze numerically the impact of the problem parameters on the optimal policy.

An understanding of optimal investment policies could provide useful insights into regulatory policies that can incentivize market participants to act in a socially beneficial manner. For example, majority of electric utility companies in the U.S. are regulated monopolies, and their investments have to be approved by a regulatory agency. The electricity prices are chosen by the regulatory agency so as to cover supplier (variable) costs, and to provide a fixed allowed return on investment (ROI) to utility companies. The allowed ROI on an asset investment is typically calculated by amortizing the cost of the asset equally over the asset's lifespan, i.e. uniform amortization. However, it is not clear that uniform amortization provides the right incentives to utility companies. Our characterization of optimal investment strategies will serve as a basis for our future work on amortization policies for both conventional and distributed generation assets.

This work is related to the literature on optimal single asset investment by Arrow [5] and Rogerson [6]. In their models, since the (social planner's) optimization problem is intertemporally separable (under the assumption that demands are strictly increasing over time), there is a simple myopic optimal policy under which the social planner procures the minimum capacity needed to meet the demand in each time period. However, these results do not extend to our two-asset investment problem, because of the complicated tradeoff between investments in the two assets.

Closer to the present paper, there is another related literature on multi-resource investment strategies [7]; for a survey, see [8]. The settings of existing works in this literature are quite different from that of this work. For example, a related body of works explore the investment strategies for flexible resources (that can be deployed to meet different types of demands) [9], [10], [11], while in our setting, there is only one type of demand, and the "nonconventional" (renewable) resource has zero variable cost.

The rest of the paper is organized as follows. In Section II, we formulate a model for the two asset investment problem and briefly review related results. In Section III, we characterize the optimal investment policy. In Section IV, we utilize our characterization to analyze numerically the impact of system parameters on the optimal transition time. In Section V, we make some concluding remarks, and briefly discuss our ongoing work on optimal amortization policies.

II. Model Formulation

In this section, we will first formulate the model considered in this paper, and then discuss some major differences between our model and existing ones. We consider a social planner that makes investments in both conventional generation assets and distributed generation assets. At each period t, the social planner makes an investment expenditure $h \cdot i_t$ in conventional generation assets, where h is the (time-invariant) price of conventional generation assets, and i_t is the conventional generation capacity installed at period t. At period t, the social planner can also choose to invest in distributed generation assets, of which the price is assumed to be $r_t = r\eta^{t-1}$ for some $\eta \in (0,1)$. At period t, let s_t denote the installed distributed generation capacity, and therefore, the cost associated with investment in distributed generation is $r_t \cdot s_t$.

The conventional generation asset i_t has a useful life of T_c periods and its productive capacity at time $t+\tau$ is given by $x_\tau \cdot i_t$. We assume that $1 = x_1 \geq x_2 \geq \ldots \geq x_{T_c} > 0$. At period t, the available productive capacity k_t is given by:

$$k_t = \sum\nolimits_{j=1}^{\min\{T_c,t\}} x_j \cdot i_{t-j+1}.$$

Let $i \triangleq (i_1, i_2, \ldots)$ denote the vector of installed conventional (generation) capacities and $k := (k_1, k_2, k_3, \ldots)$ denote the vector of available conventional (generation) capacities. We will use matrix K to denote the depreciation matrix for capacity:

$$K := \begin{pmatrix} x_1 & 0 & 0 & \ldots \\ x_2 & x_1 & 0 & \ldots \\ x_3 & x_2 & x_1 & \ldots \\ \vdots & \ddots & \ddots & \ddots \end{pmatrix}.$$

Note that $k = Ki$. We assume that there is a time-invariant unit cost w associated with the usage of conventional generation in every period.

The renewable generation asset s_t has a useful life of T_s periods and its productive capacity at time $t+\tau$ is given by $y_\tau \cdot s_t$. We assume that $1 = y_1 \geq y_2 \geq \ldots \geq y_{T_s} > 0$. Then, at period t, the available distributed generation capacity is:

$$\ell_t = \sum\nolimits_{j=1}^{\min\{T_s,t\}} y_j \cdot s_{t-j+1}.$$

Let $s := (s_1, s_2, \ldots)$ denote the vector of investments on distributed generation assets and $\ell := (\ell_1, \ell_2, \ell_3, \ldots)$ denote the vector of available distributed generation capacities. We will use L to denote the depreciation matrix for distributed generation capacity:

$$L := \begin{pmatrix} y_1 & 0 & 0 & \ldots \\ y_2 & y_1 & 0 & \ldots \\ y_3 & y_2 & y_1 & \ldots \\ \vdots & \ddots & \ddots & \ddots \end{pmatrix}.$$

Note that $\ell = Ls$. We assume that the variable cost of using distributed generation is zero. We finally note that both matrices K and L are invertible, because they are lower-triangular with nonzero diagonal entries.

We assume that consumers have an inelastic demand for electricity $\bar{q}_t$ at period t. Let $\bar{q} := (\bar{q}_1, \bar{q}_2, \ldots)$ denote the vector of consumer demands. The inelastic demand can be satisfied by using conventional or distributed generation capacity. We assume that, at each period t, consumers can use up to a quantity $\alpha\bar{q}_t$ of distributed generation capacity, where the parameter $\alpha \in [0,1]$ denotes the maximum fraction of distributed generation that can be integrated into the grid. Note that α is usually less than 1, due to the fact that certain sources of distributed generation such as wind and solar are intermittent and non-dispatchable.

Let q_t denote the demand for conventional generation in period $t \geq T_c$ and let $q \triangleq (q_1, q_2, q_3, \ldots)$ denote the vector of demands for conventional generation. Since the output of conventional generation assets cannot exceed the capacity, we have $q \leq Ki$. We assume that consumers have a discount rate $\gamma \in (0,1)$ and we let $\gamma := \left(\gamma^0, \gamma^1, \gamma^2, \ldots\right)$ denote the vector of discount rates.

The social planner aims to minimize the long-term social cost, which is derived from the investment cost (on both conventional and distributed generation assets) and the variable cost associated with conventional generation:

$$\begin{aligned} \min_{q,s,i} \quad & \gamma^T \left(hi + wq + r\text{diag}(\eta)s\right) \\ \text{subject to:} \quad & \bar{q} \leq q + Ls, \\ & q \leq Ki, \\ & \alpha\bar{q} \geq Ls, \\ & q \geq 0, \\ & i \geq 0, \\ & s \geq 0. \end{aligned} \tag{1}$$

The following proposition states that the social cost minimization problem over demand and investment quantities can be converted to an equivalent linear optimization problem over capacities only.

Proposition 1. *The social cost minimization problem* (1) *is equivalent to the following problem:*

$$\begin{aligned} \min_{k,\ell} \quad & \gamma^T \left(hK^{-1}k + w\left(\bar{q} - \ell\right) + rdiag(\eta)L^{-1}\ell\right) \\ \text{subject to:} \quad & \bar{q} \leq k + \ell, \\ & \alpha\bar{q} \geq \ell, \\ & K^{-1}k \geq 0, \\ & L^{-1}\ell \geq 0. \end{aligned} \tag{2}$$

If (q,s,i) is an optimal solution to (1)*, then $k = Ki$, $\ell = Ls$ is an optimal solution to* (2)*. If (k,ℓ) is an optimal solution to* (2)*, then $q = \bar{q} - \ell$, $s = L^{-1}\ell$, $i = K^{-1}k$ is an optimal solution to* (1).

The proof of Proposition 1 is omitted as it is straightforward. It turns out that problem (2) is more convenient for analysis and hence we will focus on analyzing problem (2) in this paper. Note that problem (2) is an infinite-dimensional linear program (LP). Hence, one could compute an approximate solution by truncating the time axis and solving the finite-dimensional version of the problem efficiently using LP solvers. However, the objective of this work is to provide insights into the structural properties of the optimal solution.

We will assume that the demand is increasing over time. This assumption is reasonable given that energy consumption typically increases over time as the economy grows. This assumption was also made in related work [5], [6], [12].

Assumption 1. *We assume that demand is strictly increasing, that is, $\bar{q}_t$ is strictly increasing for all t.*

Related Works

All existing works thus far have dealt with the single-asset investment problem [5], [6], [12]. We briefly summarize the related results here. The single-asset investment problem can be obtained as a special case of our problem (2) by fixing $\ell = 0$:

$$\begin{aligned} \min_{k} \quad & \gamma^T \left(hK^{-1}k + w\bar{q} \right) \\ \text{subject to:} \quad & \bar{q} \leq k \\ & K^{-1}k \geq 0. \end{aligned} \tag{3}$$

Under the assumption that consumers' demands $\bar{q}$ is increasing, the optimal solution to this problem is given by $k = \bar{q}$. Note that this solution is feasible, i.e., $K^{-1}\bar{q} \geq 0$, due to the fact that $\bar{q}$ is increasing. For the two-asset problem (2), even if demand is strictly increasing, however, the investment policy becomes a nontrivial sequential decision making problem, because of the tradeoff between investment in conventional and distributed generation assets. In general, an optimal policy should look ahead into future demands and prices.

III. Optimal Solution

In this section, we provide a full characterization of an optimal solution to problem (2). We start by providing some notations (associated with the characterized optimal policy) in Section III-A, and then introduce the characterization of the optimal solution in Section III-B.

A. Transition Times

For the rest of the paper, quantities with a superscript asterisk are associated with the optimal policy to be characterized in Section III-B. Under this optimal policy, let T_1 denote the first time period at which the social planner invests in distributed generation assets, i.e.,

$$T_1 \triangleq \min_{t \geq 1} \left\{ t : s_t^* > 0 \right\},$$

T_2 denote the first time period at which the distributed generation assets reaches its limit, i.e.,

$$T_2 \triangleq \min_{t \geq 1} \left\{ t : \ell_t^* \geq \alpha \bar{q}_t \right\},$$

and $T_3 \geq T_2$ be the first time period at which the social planner starts to invest in conventional generation assets so as to maintain the minimum conventional capacity $(1-\alpha)\bar{q}_t$, i.e.,

$$T_3 \triangleq \min_{t \geq T_2} \left\{ t : i_t^* > 0 \right\}.$$

We next express the quantities defined above in terms of problem parameters. Notations defined in the following will be useful in Section III-B. For an $m \times n$ matrix $A = (a_{ij})$, let $\mathcal{I} = [i_1, i_2]$ and $\mathcal{J} = [j_1, j_2]$ where $1 \leq i_1 \leq i_2 \leq m$ and $1 \leq j_1 \leq j_2 \leq n$. We denote by $A_{\mathcal{I}\mathcal{J}}$ the $(i_2 - i_1 + 1) \times (j_2 - j_1 + 1)$ submatrix of A with its (s,t) entry equal to $a_{(i_1+s-1)(j_1+t-1)}$. Similarly, for an $m \times 1$ vector $v = (v_i)$, we denote by $v_{\mathcal{I}}$ the subvector of v with s entry equal to v_{i_1+s-1}.

Suppose that the social planner invests (the minimum amount) in only conventional generation assets from period 1 to $t-1$, and starts to invest in distributed generation assets in period t. For $1 \leq t \leq t'$, we let $f_3(t', t)$ denote the difference between the conventional generation capacity and the minimum required level of conventional generation, $(1-\alpha)\bar{q}_{t'}$, in period t':

$$\begin{aligned} & f_3(t', t) \\ \triangleq & \left(K_{[1,\infty)\times[1,t-1]} K^{-1}_{[1,t-1]\times[1,t-1]} \bar{q}_{[1,t-1]} \right)_{t'} - (1-\alpha)\, \bar{q}_{t'}. \end{aligned}$$

We define $t_3(t)$ as the first time period at which conventional generation capacity decays below the minimum required level:

$$t_3(t) = \min\{t' \geq t : f_3(t', t) < 0\}, \qquad t = 1, 2, \ldots. \tag{4}$$

It is straightforward to see that $t_3(1) = 1$. If $\alpha < 1$, then $t \leq t_3(t) < \infty$ for all t because demand $\bar{q}$ is strictly increasing. If $\alpha = 1$, then $t_3(t) = \infty$ for all $t > 1$.

Next, we define the first time period at which the price of distributed generation assets goes below the variable cost of conventional generation. Formally, let $f_2(t)$ denote the difference between the net rental cost of distributed generation assets and the variable cost of conventional generation, in period t

$$f_2(t) = c^{(l)} \eta^{t-1} - w, \qquad t = 1, 2, \ldots,$$

where

$$c^{(l)} \triangleq \frac{r}{\sum_{i=0}^{\infty} y_i \gamma^{i-1} \eta^{i-1}}$$

is the rental cost of distributed generation assets. Similarly, we define the rental cost of conventional generation assets as

$$c^{(k)} \triangleq \frac{h}{\sum_{i=0}^{\infty} x_i \gamma^{i-1}}.$$

We define t_2 as the first time period at which the price of distributed generation assets goes below the variable cost of conventional generation:

$$t_2 \triangleq \min_{t} \{t : f_2(t) < 0\}. \tag{5}$$

Note that t_2 always exists, because $f_2(t)$ is strictly decreasing and converges to $-w$ as t increases to infinity.

$$f_1(t,t') = \begin{cases} \sum_{j=1}^{\min\{t_2,t_3(t)\}-t'} x_j\gamma^{j-1}\left(c^{(k)}+w-c^{(l)}\eta^{j+t'-2}\right) + \sum_{j=\min\{t_2,t_3(t)\}-t'+1}^{t_3(t)-t'} x_j\gamma^{j-1}c^{(k)}, & \text{if } t_3(t)>t, \\ c^{(k)}+w-c^{(l)}\eta^{t'-1}, & \text{if } t_3(t)=t. \end{cases} \quad (6)$$

B. Optimal Investment Policy

We now introduce the main result of this paper, in the following theorem.

Theorem 1. *Suppose that Assumption 1 holds, and at least one of the following two conditions is satisfied:*

(a) $c^{(k)} > c^{(l)} - w$,

(b) *there exists some positive integer* $t_1 \geq 2$ *such that*

$$f_1(t_1,t_1) > 0 \geq f_1(t_1,t_1-1),$$

where the mapping $f_1 : \mathbb{Z}_+ \times \mathbb{Z}_+ \to \mathbb{R}$ *is defined in* (6).

An optimal solution to problem (2) *is given by:*

$$k^* = \begin{pmatrix} \bar{q}_{\mathcal{A}} \\ K_{\mathcal{B},\mathcal{A}}K_{\mathcal{A},\mathcal{A}}^{-1}\bar{q}_{\mathcal{A}} \\ K_{\mathcal{C},\mathcal{A}}K_{\mathcal{A},\mathcal{A}}^{-1}\bar{q}_{\mathcal{A}} \\ (1-\alpha)\bar{q}_{\mathcal{D}} \end{pmatrix}, \qquad \ell^* = \begin{pmatrix} 0_{\mathcal{A}} \\ \bar{q}_{\mathcal{B}} - K_{\mathcal{B},\mathcal{A}}K_{\mathcal{A},\mathcal{A}}^{-1}\bar{q}_{\mathcal{A}} \\ \alpha\bar{q}_{\mathcal{C}} \\ \alpha\bar{q}_{\mathcal{D}} \end{pmatrix}$$

where

$$\mathcal{A} \triangleq [1, T_1 - 1], \qquad \mathcal{B} \triangleq [T_1, T_2 - 1],$$

$$\mathcal{C} \triangleq [T_2, T_3 - 1] \qquad \mathcal{D} = [T_3, \infty),$$

and[1]

$$T_1 = t_1, \qquad T_2 = \min\{t_3(t_1), t_2\}, \qquad T_3 = t_3(t_1),$$

where $t_1 = 1$ *if condition (a) holds; otherwise,* t_1 *is defined by condition (b).* ■

The proof of Theorem 1 is deferred to Appendix A. We will derive a cleaner necessary and sufficient condition for the conditions (required by Theorem 1) to hold, in Proposition 2. These conditions are indeed pretty mild, and are usually satisfied in reasonable parameter settings (cf. the numerical results in Section IV). If the conditions required by Theorem 1 hold, then there exists an optimal policy under which

1) For $t \in [1, T_1 - 1]$, invest in only conventional generation assets such that $\bar{q}_t = k_t$.
2) For $t \in [T_1, T_2 - 1]$, invest in only distributed generation assets such that $\bar{q}_t = \ell_t + k_t$.
3) For $t \in [T_2, T_3 - 1]$, invest in only distributed generation assets such that $\alpha\bar{q}_t = \ell_t$ (but it is possible that $\bar{q}_t < \ell_t + k_t$).
4) For $t \geq T_3$, invest in both types of assets such that $\alpha\bar{q}_t = \ell_t$ and $(1-\alpha)\bar{q}_t = k_t$.

[1]Interpretations of T_1, T_2 and T_3 are provided in the beginning of Section III-A.

For a special case with $\alpha = 1$, since demand can be completely satisfied using distributed generation, the optimal policy will eventually invest only in distributed generation assets. For this case, we have $T_3 = \infty$ (cf. the discussion after Eq. (4)).

Proposition 2. *Suppose that Assumption 1 holds. Condition (a) or (b) in Theorem 1 holds if and only if:*

$$h \in \left(\bigcup_{t=2}^{\infty} (\underline{f}(t), \overline{f}(t)]\right) \bigcup (\underline{f}(1), \infty)$$

where the mapping $\overline{f} : \mathbb{Z}_+/\{1\} \to \mathbb{R}$ *is defined in* (7)*, and* $\underline{f} : \mathbb{Z}_+ \to \mathbb{R}$ *is defined in* (8) *(located at the top of the next page).*

The proof of Proposition 2 is given in Appendix C, where we also show a nice monotonicity property for mappings $\overline{f}$ and $\underline{f}$: if $\alpha < 1$, then $0 \leq \underline{f}(t) \leq \overline{f}(t) \leq \underline{f}(t-1)$ for all $t \geq 2$, and otherwise $0 \leq \underline{f}(t) \leq \overline{f}(t) = \underline{f}(t-1)$ for all $t \geq 2$.

IV. NUMERICAL EXAMPLES

In this section, we illustrate numerically the impact of the problem parameters on the optimal transition times. The numerical results also demonstrate that the conditions required by Theorem 1 hold in most of the cases.

We start by describing the parameters that are used in our numerical examples. Although our model allows for arbitrary increasing consumer demands and asset depreciation patterns, numerical results presented in this section is for a simple setting, where consumer demand increases linearly over the first 30 years, and then increases exponentially at rate ξ,

$$\bar{q}_t = \begin{cases} \dfrac{t}{30}\bar{q}_0, & 1 \leq t \leq 30, \\ \bar{q}_0\xi^{t-30}, & t > 30, \end{cases}$$

where $\bar{q}_0 = 1$ and $\xi = 1.02$. We assume one hoss shay depreciation for both conventional and renewable assets, that is, $x_1 = x_2 = \ldots = x_{T_c} = 1$ and $y_1 = y_2 = \ldots = y_{T_s} = 1$. Unless otherwise stated, parameters in our numerical results are set up as follows,

$$\begin{aligned} T_c &= 20, & T_s &= 20, \\ h &= 1.0, & w &= 0.06, \\ r &= 10.0, & \eta &= 0.95, \\ \alpha &= 0.30, & \gamma &= 0.98. \end{aligned}$$

In this setting, both the conventional and renewable assets have a lifespan of $T_c = T_2 = 20$ years. The variable cost of conventional generation $w = \$0.06/kWh$ and the price of conventional assets $h = 1.0$. Here, h has been chosen so that

$$\overline{f}(t) = \begin{cases} \left(\sum_{j=1}^{\min\{t_2, t_3(t)\}-(t-1)} x_j \gamma^{j-1} \left(c^{(l)} \eta^{j+t-3} - w \right) \right) \left(\sum_{j=1}^{\infty} x_j \gamma^{j-1} \right) \Big/ \left(\sum_{j=1}^{t_3(t)-(t-1)} x_j \gamma^{j-1} \right), & \text{if } t_3(t) > t, \\ \left(c^{(l)} \eta^{t-2} - w \right) \sum_{j=1}^{\infty} x_j \gamma^{j-1}, & \text{if } t_3(t) = t. \end{cases} \quad (7)$$

$$\underline{f}(t) = \begin{cases} \left(\sum_{j=1}^{\min\{t_2, t_3(t)\}-t} x_j \gamma^{j-1} \left(c^{(l)} \eta^{j+t-2} - w \right) \right) \left(\sum_{j=1}^{\infty} x_j \gamma^{j-1} \right) \Big/ \left(\sum_{j=1}^{t_3(t)-t} x_j \gamma^{j-1} \right), & \text{if } t_3(t) > t, \\ \left(c^{(l)} \eta^{t-1} - w \right) \sum_{j=1}^{\infty} x_j \gamma^{j-1}, & \text{if } t_3(t) = t. \end{cases} \quad (8)$$

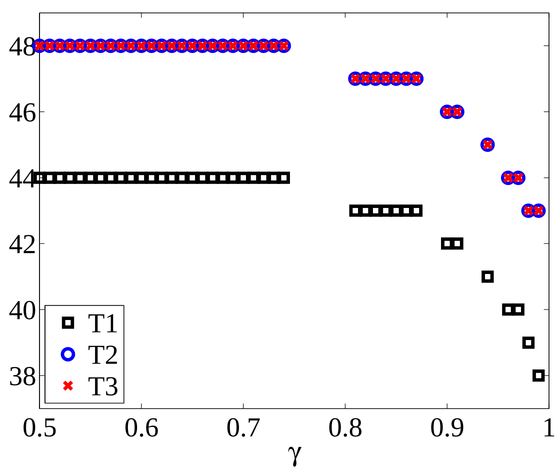

Fig. 1. Optimal transition times versus the discount rate γ.

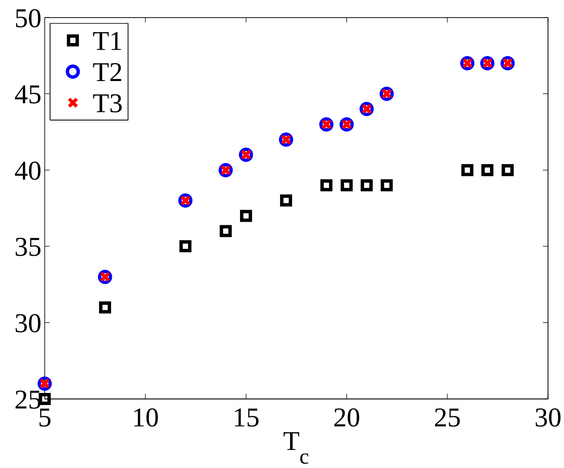

Fig. 2. Optimal transition times versus the lifespan of conventional asset T_c.

the rental price of conventional asset $c^{(k)}$ approximately equals the variable cost w (in the real world, fixed asset costs typically comprise about half of total electricity generation costs). We assume a high initial price of renewable assets ($r = 10.0$), which decays by 5% per year.

Fig. 1 plots the optimal transition times (T_1, T_2, T_3) (characterized in Theorem 1) versus the discount rate γ. There are only a few missing points in the plot at which conditions required by Theorem 1 are not satisfied, i.e., Theorem 1 holds for a wide range of values of γ. We note that $T_2 = T_3$ for all values of γ, i.e., we have $\bar{q}_t = \ell_t + k_t$ for all t (cf. the discussion following Theorem 1). This is the case observed in all our numerical results, and $T_2 < T_3$ occurs only at extremes cases (e.g., when the initial price of distributed generation assets r is extremely high).

We observe from Fig. 1 that both the optimal transition times T_1 and T_2 decrease with the discount rate. In fact, this has to be the case, if $T_2 = T_3$. The monotonicity of T_1 can be proved by using the fact that $f_1(t, t')$ (defined in Eq. (6)) decreases in γ. T_2 (i.e., T_3) is also monotonically decreasing in γ because $T_3 = t_3(T_1)$ and the mapping $t_3(\cdot)$ is increasing.

Fig. 2 plots the optimal transition times (T_1, T_2, T_3), as the lifespan of conventional asset T_c increases from 5 years to 30 years. It can be seen that conditions required by Theorem 1 hold for most values of T_c, as there are only a handful of missing points in the plot. As the lifespan of conventional asset T_c increases, conventional asset becomes relatively cheaper and more valuable, which in turn increases the optimal transition time T_1. T_2 (i.e., T_3) also increases in T_c, due to the fact that $T_3 = t_3(T_1)$ and the mapping $t_3(\cdot)$ is increasing.

Fig. 3 plots the optimal transition times (T_1, T_2, T_3) versus the lifespan of renewable asset T_s. Again, we see that the conditions in Theorem 1 hold for most values of T_s. As the lifespan of distributed asset T_s increases, distributed asset becomes relatively cheaper and more valuable, and the optimal transition time T_1 therefore decreases.

V. Conclusion and Future Work

We formulate a linear programming problem to study the tradeoff between investments in conventional and renewable generation assets. Our model accounts for a variety of factors including the depreciation of existing capacities, future demands for electricity, as well as future investment prices and operating costs. We characterize the optimal investment policy for a social planner, who seeks to minimize the long-term discounted costs (associated with both the procurement and the usage of conventional and distributed generation assets). In particular, under mild conditions on problem parameters, we

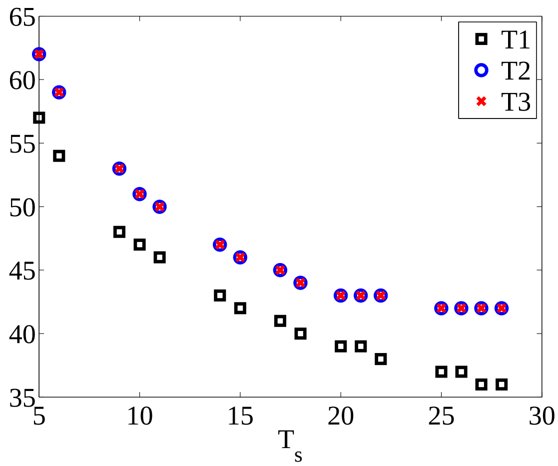

Fig. 3. Optimal transition times versus the lifespan of renewable asset T_s.

provide closed-form expressions for the optimal time period at which investment should switch from conventional assets to renewable assets. Numerical examples illustrate that these conditions (on the problem parameters) are usually satisfied in reasonable parameter settings.

The model and results presented in this paper can serve as a basis for our future work on optimal amortization policies for conventional and distributed assets. In practice, it is unlikely to have a social planner who can directly implement the optimal investment policy. Investments in conventional and renewable generation assets are likely to be driven by market mechanisms, possibly under the governance of regulatory agents such as a public utilities commission. Distributed renewable generation assets would be purchased by consumers to reduce their electricity bills, while conventional generation assets are typically procured by regulated utility companies. Regulatory policies (e.g., asset cost allocation policies) could have an impact on the behavior of market participants (utility companies and electricity consumers).

For instance, in rate-of-return (RoR) regulation, the utility company sets prices to recover its variable operating costs, asset depreciation charges, as well as interest on its asset investments. The depreciation schedule would have an impact on the electricity prices set by the utility company. The electricity prices would, in turn, influence the consumers' decisions on how and when to purchase distributed generation assets. The characterization of a (socially) optimal investment strategy presented in this paper would be useful for investigating optimal depreciation schedules that can implement the optimal investment strategy at a market equilibrium (among regulated utility companies and electricity consumers).

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Appendix A
Proof of Theorem 1

Before proving the theorem, we first argue a simple fact that conditions (a) and (b) cannot simultaneously hold. Indeed, condition (a) implies that $f_1(t_1, t_1 - 1) \geq 0$, which contradicts condition (b).

The main idea of the proof is to show that the solution characterized in this theorem is legitimate, because it satisfies the Karush-Kuhn Tucker (KKT) conditions for some nonnegative Lagrange multipliers. Since problem (2) is a linear program, the KKT conditions are necessary and sufficient. Recall that $t_3(t) \geq t$. Moreover, from the definition of t_1 and t_2, we have $t_2 \geq t_1$. Therefore, there are five possible cases of possible relations among $(t_1, t_2, t_3(t_1))$:

$$
\begin{aligned}
&1 < t_1 < t_2 < t_3(t_1), \\
&1 < t_1 = t_2 < t_3(t_1), \\
&1 < t_1 < t_3(t_1) \leq t_2, \\
&1 < t_1 = t_3(t_1) \leq t_2, \\
&1 = t_1 = t_3(t_1).
\end{aligned}
$$

As the KKT conditions for each of the above cases are different, a separate discussion is required for each case. Nevertheless, the techniques for all the proofs are similar. Therefore, we will only present the proof for the most complicated case with $1 < t_1 < t_2 < t_3(t_1)$. In step 1, we first define some notations and introduce the Lagrange multipliers associated with the proposed solution. In step 2, we argue that the proposed solution for (k, ℓ) in the Theorem statement is feasible. In step 3, we argue that the proposed solution for (k, ℓ) in the Theorem statement and the given Lagrange multipliers satisfy the complementary slackness conditions. In step 4, we show that the proposed solution for (k, ℓ) in the Theorem statement and the given Lagrange multipliers satisfy the first-order optimality conditions. Finally, in step 5, we show that the given Lagrange multipliers are nonnegative.

Step 1: We let $(\lambda, \mu, \theta, \phi)$ be the Lagrange multipliers associated with the constraints in (2) such that the complementary slackness conditions are given by

$$
\begin{aligned}
\lambda \circ (\bar{q} - k - \ell) &= 0, \\
\mu \circ (\ell - \alpha \bar{q}) &= 0, \\
\theta \circ K^{-1} k &= 0, \\
\phi \circ L^{-1} \ell &= 0.
\end{aligned}
$$

We show that the solution (k, ℓ) given in the Theorem statement satisfy the KKT conditions with the following Lagrange multipliers:

$$
\begin{aligned}
\theta_{\mathcal{A}} &= \mu_{\mathcal{A}} = 0, \\
\phi_{\mathcal{B}} &= \mu_{\mathcal{B}} = 0, \\
\phi_{\mathcal{C}} &= \lambda_{\mathcal{C}} = 0, \\
\theta_{\mathcal{D}} &= \phi_{\mathcal{D}} = 0,
\end{aligned}
$$

and

$$\lambda_{\mathcal{A}} = K_{\mathcal{A}\mathcal{A}}^{-T}\Big(c^{(k)}\left(K_{\mathcal{A}\mathcal{A}}^T\gamma_{\mathcal{A}} + K_{\mathcal{B}\mathcal{A}}^T\gamma_{\mathcal{B}} + K_{\mathcal{C}\mathcal{A}}^T\gamma_{\mathcal{C}}\right) + K_{\mathcal{B}\mathcal{A}}^T\text{diag}\left(w - c^{(l)}\eta_{\mathcal{B}}\right)\gamma_{\mathcal{B}}\Big),$$

$$\phi_{\mathcal{A}} = L_{\mathcal{A}\mathcal{A}}^T\left(\text{diag}\left(c^{(l)}\eta_{\mathcal{A}} - w\right)\gamma_{\mathcal{A}} - \lambda_{\mathcal{A}}\right),$$

$$\theta_{\mathcal{B}} = K_{\mathcal{B}\mathcal{B}}^T\left(\text{diag}\left(c^{(k)} + w - c^{(l)}\eta_{\mathcal{B}}\right)\gamma_{\mathcal{B}} + c^{(k)}K_{\mathcal{B}\mathcal{B}}^{-T}K_{\mathcal{C}\mathcal{B}}^T\gamma_{\mathcal{C}}\right),$$

$$\lambda_{\mathcal{B}} = \text{diag}\left(c^{(l)}\eta_{\mathcal{B}} - w\right)\gamma_{\mathcal{B}},$$

$$\theta_{\mathcal{C}} = c^{(k)}K_{\mathcal{C}\mathcal{C}}^T\gamma_{\mathcal{C}},$$

$$\mu_{\mathcal{C}} = \text{diag}\left(w - c^{(l)}\eta_{\mathcal{C}}\right)\gamma_{\mathcal{C}},$$

$$\lambda_{\mathcal{D}} = c^{(k)}\gamma_{\mathcal{D}},$$

$$\mu_{\mathcal{D}} = \text{diag}\left(c^{(k)} + w - c^{(l)}\eta_{\mathcal{D}}\right)\gamma_{\mathcal{D}}.$$

Step 2: It is straightforward to check that the solution (k, ℓ) given in the Theorem statement in feasible in segments $\mathcal{A}$ and $\mathcal{D}$. To see that the solution is feasible in segments $\mathcal{B}$ and $\mathcal{C}$, use the definition of T_3.

Step 3: It is straightforward to check that the complementary slackness conditions are satisfied.

Step 4: In this step, we show that the proposed Lagrange multipliers and the proposed solution (k, ℓ) given in the Theorem statement satisfy the first-order conditions. The first-order conditions are given by:

$$hK^{-T}\gamma - \lambda - K^{-T}\theta = 0$$
$$rL^{-T}\text{diag}(\eta)\gamma - w\gamma - \lambda + \mu - L^{-T}\phi = 0.$$

By writing K^{-1} and L^{-1} in terms of their sub-matrices, it is straightforward to check that the first-order conditions are satisfied.

Step 5: In this step, we show that the proposed Lagrange multipliers are nonnegative. Clearly, $\theta_{\mathcal{C}} \geq 0$ and $\lambda_{\mathcal{D}} \geq 0$. Since $w \geq c^{(l)}\eta^{t-1}$ for all $t \geq T_2$, it follows that $\mu_{\mathcal{C}} \geq 0$ and $\mu_{\mathcal{D}} \geq 0$. Similarly, since $w < c^{(l)}\eta^{t-1}$ for all $t \leq T_2 - 1$, it follows that $\lambda_{\mathcal{B}} \geq 0$. To see that $\theta_{\mathcal{B}} \geq 0$, first rewrite $\theta_{\mathcal{B}}$ as follows:

$$\theta_{\mathcal{B}} = \begin{pmatrix} K_{\mathcal{B}\mathcal{B}}^T & K_{\mathcal{C}\mathcal{B}}^T \end{pmatrix}\text{diag}\begin{pmatrix}\gamma_{\mathcal{B}} \\ \gamma_{\mathcal{C}}\end{pmatrix}\begin{pmatrix} c^{(k)} + w - c^{(l)}\eta_{\mathcal{B}} \\ c^{(k)}\end{pmatrix}.$$

By Lemma 1[2], we have that if $\theta_{T_1} \geq 0$, then $\theta_{\mathcal{B}} \geq 0$. However, $\theta_{T_1} = f_1(T_1, T_1) \geq 0$ from the definition of T_1. Hence, $\theta_{\mathcal{B}} \geq 0$. Next, we show that $\lambda_{\mathcal{A}} \geq 0$. By Lemma 2, if the vector:

$$c^{(k)}\left(K_{\mathcal{A}\mathcal{A}}^T\gamma_{\mathcal{A}} + K_{\mathcal{B}\mathcal{A}}^T\gamma_{\mathcal{B}} + K_{\mathcal{C}\mathcal{A}}^T\gamma_{\mathcal{C}}\right) + K_{\mathcal{B}\mathcal{A}}^T\text{diag}\left(w - c^{(l)}\eta_{\mathcal{B}}\right)\gamma_{\mathcal{B}}$$

is positive and decreasing, then $\lambda_{\mathcal{A}} \geq 0$. The ith entry of the first term is:

$$c^{(k)}\left(K_{\mathcal{A}\mathcal{A}}^T\gamma_{\mathcal{A}} + K_{\mathcal{B}\mathcal{A}}^T\gamma_{\mathcal{B}} + K_{\mathcal{C}\mathcal{A}}^T\gamma_{\mathcal{C}}\right)_i = c^{(k)}\sum_{j=1}^{T_3-i} x_j\gamma^{j+i-2}$$

which is decreasing in i. The ith entry of the second term is:

$$\left(K_{\mathcal{B}\mathcal{A}}^T\text{diag}\left(w - c^{(l)}\eta_{\mathcal{B}}\right)\gamma_{\mathcal{B}}\right)_i = \sum_{j=T_1-1}^{T_2-2} x_{j-i+2}\gamma^j\left(w - c^{(l)}\eta^j\right)$$

which is decreasing in i since $w - rc^{(l)}\eta^{j-1} \leq 0$ for all $T_1 \leq j \leq T_2 - 1$ and $x_{j-i+2} \geq x_{j-(i+1)+2}$. Hence, for the desired vector to be positive, it is sufficient for its last entry to be positive. The latter is true because:

$$\begin{aligned}
&\Big(c^{(k)}\left(K_{\mathcal{A}\mathcal{A}}^T\gamma_{\mathcal{A}} + K_{\mathcal{B}\mathcal{A}}^T\gamma_{\mathcal{B}} + K_{\mathcal{C}\mathcal{A}}^T\gamma_c\right) + K_{\mathcal{B}\mathcal{A}}^T\text{diag}\left(w - c^{(l)}\eta_{\mathcal{B}}\right)\gamma_{\mathcal{B}}\Big)_{T_1-1} \\
&= c^{(k)}\sum_{j=1}^{T_3-T_1+1} x_j\gamma^{j+T_1-3} + \sum_{j=T_1-1}^{T_2-2} x_{j-T_1+3}\gamma^j\left(w - c^{(l)}\eta^j\right) \\
&= \gamma^{T_1-2}\left(c^{(k)}\sum_{j=1}^{T_3-T_1+1} x_j\gamma^{j-1} + \sum_{j=2}^{T_2-T_1+1} x_j\gamma^{j-1}\left(w - c^{(l)}\eta^{j-T_1-3}\right)\right) \\
&\geq \gamma^{T_1-2}\left(c^{(k)}x_{T_3-T_1+1}\gamma^{T_3-T_1} + f_1(T_1, T_1)\right) \\
&\geq 0,
\end{aligned}$$

where the first inequality follows from the fact that $w \leq c^{(l)}\eta^{j-T_1-3}$ for all $2 \leq j \leq T_2 - T_1 + 1$ and the last inequality follows from the definition of T_1. Finally, we show that $\phi_{\mathcal{A}} \geq 0$. Rewrite $\phi_{\mathcal{A}}$ as follows:

$$\phi_{\mathcal{A}} = L_{\mathcal{A}\mathcal{A}}^T K_{\mathcal{A}\mathcal{A}}^{-T}\begin{pmatrix}\text{diag}(\gamma_{\mathcal{A}})K_{\mathcal{A}\mathcal{A}} \\ \text{diag}(\gamma_{\mathcal{B}})K_{\mathcal{B}\mathcal{A}} \\ \text{diag}(\gamma_{\mathcal{C}})K_{\mathcal{C}\mathcal{A}}\end{pmatrix}^T\begin{pmatrix} c^{(l)}\eta_{\mathcal{A}} - w - c^{(k)} \\ c^{(l)}\eta_{\mathcal{B}} - w - c^{(k)} \\ -c^{(k)}\end{pmatrix}.$$

Note that the vector $\begin{pmatrix} c^{(l)}\eta_{\mathcal{A}} - w - c^{(k)} \\ c^{(l)}\eta_{\mathcal{B}} - w - c^{(k)} \\ -c^{(k)}\end{pmatrix}$ is decreasing. Moreover, we have:

$$\begin{aligned}
&\left(\begin{pmatrix}\text{diag}(\gamma_{\mathcal{A}})K_{\mathcal{A}\mathcal{A}} \\ \text{diag}(\gamma_{\mathcal{B}})K_{\mathcal{B}\mathcal{A}} \\ \text{diag}(\gamma_{\mathcal{C}})K_{\mathcal{C}\mathcal{A}}\end{pmatrix}^T\begin{pmatrix} c^{(l)}\eta_{\mathcal{A}} - w - c^{(k)} \\ c^{(l)}\eta_{\mathcal{B}} - w - c^{(k)} \\ -c^{(k)}\end{pmatrix}\right)_{T_1-1} \\
&= \gamma^{T_1-2}\left(\sum_{j=1}^{T_2-T_1+1} x_j\gamma^{j-1}\left(c^{(l)}\eta^{j+T_1-3} - c^{(k)} - w\right) - \sum_{j=T_2-T_1+2}^{T_3-T_1+1} x_j\gamma^{j-1}c^{(k)}\right) \\
&= -\gamma^{T_1-2}f(T_1, T_1 - 1) \\
&\geq 0,
\end{aligned}$$

where the last inequality follows from the definition of T_1. By Lemma 2 and the fact that all the entries of $L_{\mathcal{A}\mathcal{A}}$ are nonnegative, it follows that $\phi_{\mathcal{A}} \geq 0$.

[2]The statements and proofs of Lemmas 1 and 2 are given in Appendix B.

Appendix B
Supplementary Results

Lemma 1. *Let M be a $N_1 \times N_2$ matrix (with $N_1 \geq N_2$) of the form:*

$$M = \begin{pmatrix} m_1 & 0 & \dots & 0 \\ m_2 & m_1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ m_{N_2} & m_{N_2-1} & \dots & m_1 \\ m_{N_2+1} & m_{N_2} & \dots & m_2 \\ \vdots & \ddots & \ddots & \vdots \\ m_{N_1} & m_{N_1-1} & \dots & m_{N_1-N_2+1} \end{pmatrix}$$

where $m_1 \geq m_2 \geq m_3 \geq \ldots \geq m_N \geq 0$ and let $\beta = (\beta^0, \beta^1, \beta^2, \ldots)$ where $0 < \beta < 1$.

(a) *For any increasing vector v, if $\left(M^T diag(\beta)v\right)_1 \geq 0$, then $M^T diag(\beta)v \geq 0$. If $(M^T diag(\beta)v)_i < 0$, then $(M^T diag(\beta)v)_i \leq (M^T diag(\beta)v)_{i+1}$.*

(b) *For any decreasing vector v, if $\left(M^T diag(\beta)v\right)_{N_2} \geq 0$, then $M^T diag(\beta)v \geq 0$. Moreover, $M^T diag(\beta)v$ is decreasing.*

Proof: We only give the proof for part (a). The proof for part (b) is similar. If $v \geq 0$, then $M^T\text{diag}(\beta)v \geq 0$ by definition. Hence, suppose that $v_i < 0$ for some i. Since v is increasing, there exists some T such that $v_i < 0$ for all $i \leq T$ and $v_i \geq 0$ otherwise. The ith component of $M^T\text{diag}(\beta)v$ is given by:

$$\begin{aligned}\left(M^T\text{diag}(\beta)v\right)_i &= \sum_{j=1}^{N_1-i+1} m_j \beta^{j+i-2} v_{j+i-1} \\ &= \sum_{j=i}^{T} m_{j-i+1}\beta^{j-1}v_j + \sum_{j=T+1}^{N_1} m_{j-i+1}\beta^{j-1}v_j.\end{aligned}$$

Suppose $i > T$. Then the first sum is zero. The second sum is positive since the summands are positive for all $T+1 \leq j \leq N_1$. Hence, $\left(M^T\text{diag}(\beta)v\right)_i \geq 0$ for all $i > T$.

Suppose $i \leq T$. We will show that both sums are increasing with respect to i and hence $\left(M^T\text{diag}(\beta)v\right)_1 \geq 0$ implies that $\left(M^T\text{diag}(\beta)v\right)_i \geq 0$ for all $1 \leq i \leq T$. The first sum is increasing with respect to i since:

$$\begin{aligned}\sum_{j=i}^{T} m_{j-i+1}\beta^{j-1}v_j &\leq \sum_{j=i}^{T-1} m_{j-i+1}\beta^{j-1}v_j \\ &\leq \sum_{j=i}^{T-1} m_{j-i+1}\beta^{j}v_{j+1} \\ &= \sum_{j=i+1}^{T} m_{j-(i+1)+1}\beta^{j-1}v_j,\end{aligned}$$

where the first inequality follows from the fact that $m_{T-i+1}\beta^{T-1}v_T < 0$, and the second inequality follows from the fact that $m_{j-i+1}v_{j+1} < 0$ for $i \leq j \leq T-1$. The second sum is increasing with respect to i since:

$$\sum_{j=T+1}^{N_1} m_{j-i+1}\beta^{j-1}v_j \leq \sum_{j=T+1}^{N_1} m_{j-(i+1)+1}\beta^{j-1}v_j$$

where the inequality follows from the fact that $\beta^{j-1}v_j \geq 0$ and $m_{j-i+1} \leq m_{j-(i+1)+1}$. Hence, both sums are increasing with respect to i.

Finally, if $(M^T\text{diag}(\beta)v)_i < 0$, then $i \leq T$, and we have shown that $(M^T\text{diag}(\beta)v)_i \leq (M^T\text{diag}(\beta)v)_{i+1}$ in this case. ■

Lemma 2. *Let M be a $N \times N$ lower-triangular Toeplitz matrix of the form:*

$$M = \begin{pmatrix} m_1 & 0 & 0 & \dots & 0 \\ m_2 & m_1 & 0 & \dots & 0 \\ m_3 & m_2 & m_1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ m_N & m_{N-1} & m_{N-2} & \dots & m_1 \end{pmatrix}$$

where $m_1 \geq m_2 \geq m_3 \geq \ldots \geq m_N \geq 0$ and $m_1 > 0$. Let v be a positive and decreasing vector. Then $M^{-T}v \geq 0$.

Proof: Note that since M is lower-triangular with non-zero diagonal entries, it is invertible. We prove the lemma by induction on the size of the matrix N. Clearly, the lemma holds for $N = 1$. Suppose the lemma holds for some $N = n$. We show that it must also hold for $N = n+1$. Let the vector $\tilde{m}$ be defined by $\tilde{m} = (m_{n+1}, m_n, \ldots, m_1)$. Partition the $(n+1) \times (n+1)$ matrix as follows:

$$M_{[1:n+1]\times[1:n+1]} = \begin{pmatrix} M_{[1:n]\times[1:n]} & 0 \\ \tilde{m}_{[1:n]}^T & m_1 \end{pmatrix}.$$

It follows that

$$\begin{aligned}&M^{-T}_{[1:n+1]\times[1:n+1]}v \\ &= \begin{pmatrix} M^{-T}_{[1:n]\times[1:n]} & -\frac{1}{m_1}M^{-T}_{[1:n]\times[1:n]}\tilde{m}_{[1:n]} \\ 0 & 1/m_1 \end{pmatrix}\begin{pmatrix} v_{[1:n]} \\ v_{n+1} \end{pmatrix} \\ &= \begin{pmatrix} M^{-T}_{[1:n]\times[1:n]}\left(v_{[1:n]} - \frac{v_{n+1}}{m_1}\tilde{m}_{[1:n]}\right) \\ \frac{v_{n+1}}{m_1} \end{pmatrix}.\end{aligned}$$

Note that $v_{[1:n]} - (v_{n+1}/m_1)\tilde{m}_{[1:n]}$ is decreasing since $\tilde{m}_{[1:n]}$ is increasing. Moreover, $v_{[1:n]} - (v_{n+1}/m_1)\tilde{m}_{[1:n]}$ is also positive since $\tilde{m}_{[1:n]}/m_1 \leq 1$ and v is decreasing. Hence, by the induction hypothesis, $M^{-T}_{[1:n]\times[1:n]}\left(v_{[1:n]} - (v_{n+1}/m_1)\tilde{m}_{[1:n]}\right) \geq 0$. Since $v_{n+1}/m_1 \geq 0$, we have that $M^{-T}_{[1:n+1]\times[1:n+1]}v \geq 0$. ■

Appendix C
Proof of Proposition 2

Proof: Our proof consists of four steps. In step 1, we show that condition (a) in Theorem 1 holds if and only if $h \in \left(\underline{f}(1), \infty\right)$. In step 2, we show that condition (b) in Theorem 1 holds if and only if $h \in \bigcup_{t=2}^{\infty}\left(\underline{f}(t), \overline{f}(t)\right]$. In step 3, we show that $\overline{f}(t) \leq \underline{f}(t-1)$ for all $t \geq 2$. In step 4, we show that $\underline{f}(t) \leq \overline{f}(t)$ for all $t \geq 2$. It is straightforward to observe that $\overline{f}(t) \geq 0$ for all $t \geq 2$, so this observation together with steps 3 and 4 implies that $0 \leq \underline{f}(t) \leq \overline{f}(t) \leq \underline{f}(t-1)$ for all $t \geq 2$. Finally, it is straightforward to see that, if $\alpha = 1$, then $\underline{f}(t) \leq \overline{f}(t) = \underline{f}(t-1)$ for all $t \geq 2$.

Step 1: It is easy to see that condition (a) in Theorem 1 is equivalent to $h \geq \underline{f}(1)$.

Step 2: It is easy to see that condition (b) in Theorem 1 is equivalent to $h \in \bigcup_{t=2}^{\infty} \left(\underline{f}(t), \overline{f}(t)\right]$ by noting that $f_1(t,t) > 0 \geq f_1(t,t-1)$ is equivalent to:

$$\underline{f}(t) < h \leq \overline{f}(t).$$

Step 3: In this step, we show that $\overline{f}(t) \leq \underline{f}(t-1)$ for all $t \geq 2$. We prove by contradiction. Suppose $\overline{f}(t) > \underline{f}(t-1)$. Then there exists h such that:

$$\overline{f}(t) > h > \underline{f}(t-1).$$

These inequalities can be rewritten as:

$$\begin{aligned} f_1(t,t-1) &< 0 \\ f_1(t-1,t-1) &> 0. \end{aligned}$$

We treat each of the following cases separately:

(a) $t_3(t) = t$ and $t_3(t-1) = t-1$.

(b) $t_3(t) = t$ and $t_3(t-1) > t-1$.

(c) $t_3(t) > t$ and $t_3(t-1) = t-1$.

(d) $t_3(t) > t$ and $t_3(t-1) > t-1$.

We present the proof for the remaining cases in the following four steps.

Step 3a: Suppose $t_3(t) = t$ and $t_3(t-1) = t-1$. It is straightforward to see that $\overline{f}(t) = \underline{f}(t-1)$ which is a contradiction.

Step 3b: Suppose $t_3(t) = t$ and $t_3(t-1) > t-1$. Since $t-1 < t_3(t-1) \leq t_3(t) = t$, we have $t_3(t-1) = t$. Since $f_1(t,t-1) < 0$, we have $c^{(k)} + w - c^{(l)}\eta^{t-2} < 0$. It is easy to see that $f_1(t,t-1) = f_1(t-1,t-1)$ which is a contradiction.

Step 3c: Suppose $t_3(t) > t$ and $t_3(t-1) = t-1$. Since $f_1(t-1,t-1) > 0$, we have $c^{(k)} + w - c^{(l)}\eta^{t-2} > 0$, which implies that $f_1(t,t-1) > 0$, which is a contradiction.

Step 3d: Suppose $t_3(t) > t$ and $t_3(t-1) > t-1$. If $t_2 \leq t_3(t-1) \leq t_3(t)$, then it follows that $f_1(t,t-1) > f_1(t-1,t-1)$ which is a contradiction. If $t_3(t-1) < t_2 \leq t_3(t)$, then $f_1(t-1,t-1) > 0$ implies that $x_j \gamma^{j-1} \left(c^{(k)} + w - c^{(l)}\eta^{j+t-3}\right) \geq 0$ for $j \geq t_3(t-1) - (t-1)$, which implies that $f_1(t,t-1) \geq f_1(t-1,t-1)$, which is a contradiction.

Step 4: In this step, we show that $\underline{f}(t) \leq \overline{f}(t)$ for all $t \geq 2$. The case where $t_3(t) = t$ is easy. Hence, we only prove the case where $t_3(t) > t$. We prove this case by contradiction. Suppose $\underline{f}(t) > \overline{f}(t)$. Then there exists a h such that:

$$\underline{f}(t) > h > \overline{f}(t).$$

These inequalities can be rewritten as:

$$\begin{aligned} f_1(t,t) &< 0 \\ f_1(t,t-1) &> 0. \end{aligned}$$

However, it is easy to write $f_1(t,\cdot)$ in matrix form such that Lemma 1 applies. Since $f_1(t,t) < 0$, we have that $f_1(t,t) \geq f_1(t,t-1)$ which is a contradiction. ■

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