Real-Time Deferrable Load Control: Handling the Uncertainties of Renewable Generation*

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ABSTRACT

Real-time demand response is potential to handle the uncertainties of renewable generation. It is expected that a large number of deferrable loads, including electric vehicles and smart appliances, will participate in demand response in the future. In this paper, we propose a decentralized algorithm that reduces the tracking error between demand and generation, by shifting the power consumption of deferrable loads to match the generation in real-time. At each time step within the control window, the algorithm minimizes the expected tracking error to go with updated predictions on demand and generation. It is proved that the root mean square tracking error vanishes as control window expands, even in the presence of prediction errors.

Keywords

smart grid, deferrable load control, demand response

1. INTRODUCTION

Real-time demand response seeks to induce dynamic load management in response to power supply conditions, e.g., by shifting the power consumption of deferrable loads, including plug-in electric vehicles and smart appliances, to compensate for the random fluctuations in renewable generation.

Work on deferrable load control falls into two categories: direct load control (DLC) [6, 7, 9, 11], which obtains reliable demand-side response and is used mainly for household loads; and pricing mechanisms [1, 2, 10], which allow consumers to implement their own control mechanisms and are used mainly for large industrial loads. The work summarized here focuses on DLC, more specifically decentralized algorithms to coordinate a large number of deferrable loads.

This extended abstract summarizes the recent work [8] where we propose a decentralized real-time deferrable load control algorithm to reduce the tracking error between generation and demand. At each time step within the control window, the algorithm minimizes the expected tracking error to go with updated predictions on future demand and generation. The algorithm has provable performance guarantees—the root mean square tracking error approaches the optimal value as control window expands. Finally, we compare the expected tracking error achieved by the proposed algorithm with that achieved by the optimal open-loop control, and show that the improvement over open-loop control increases as control window expands.

The impact of prediction error on the proposed algorithm is studied explicitly and analytically. Most prior works study prediction error empirically [4, 5], and works with analytic results mostly take a "worst-case" perspective [3, 12], as opposed to the "average-case" perspective of the work here.

2. PROBLEM FORMULATION

The work summarized here studies real-time algorithms for scheduling deferrable loads to compensate for the random fluctuations in renewable generation. Throughout, we consider a discrete-time model over a finite time horizon, which is divided into T time slots $1, \ldots, T$ of equal length.

Assume N deferrable loads $1, \ldots, N$ arrive over time, each requiring certain amount of electricity by certain deadline. Without loss of generality, assume that loads $1, \ldots, N(t)$ arrive before or at time t, for $t = 1, \ldots, T$. Define $N(0) := 0$. Let $d_n(t)$ denote the power consumption of load n at time t, g denote the generation, and e denote the tracking error between demand and generation.

The optimal deferrable load control (ODLC) problem seeks to minimize the square tracking error (1a), subject to constraints (1c)–(1d) on deferrable loads, as given below.

$$\textbf{ODLC:} \quad \min \sum_{t=1}^{T} e^2(t) \tag{1a}$$

$$\text{over} \quad d_n(t), e(t), \quad \forall n, t$$

$$\text{s.t.} \quad e(t) = \sum_{n=1}^{N} d_n(t) - g(t), \quad \forall t; \tag{1b}$$

$$\underline{d}_n(t) \le d_n(t) \le \overline{d}_n(t), \quad \forall n, t; \tag{1c}$$

$$\sum_{t=1}^{T} d_n(t) = D_n, \quad \forall n, \tag{1d}$$

where $\underline{d}_n$, $\overline{d}_n$ and D_n are given constants for all n. Constraints (1c) impose power consumption to be bounded and constraints (1d) impose energy consumption to be fixed.

Finally, deferrable loads arrive randomly over time. Let

$$a(t) := \sum_{n=N(t-1)+1}^{N(t)} D_n, \qquad t = 1, \ldots, T$$

denote the energy request of deferrable loads that arrive at time t, and assume $\{a(t)\}_{t=1}^{T}$ to be a sequence of uncorrelated random variables with mean λ and variance s^2. Fur-

*This extended abstract summarizes recent work presented at eEnergy conference. The full version of the paper can be found in [8].

ther, let $A(t) := \sum_{\tau=t+1}^{T} a(\tau)$ denote the total energy requested after time t.

At time t, real-time algorithms know 1) present deferrable loads, i.e., $\underline{d}_n$, $\overline{d}_n$, and D_n for $n \leq N(t)$; 2) expectation $\mathbb{E}(A(t))$ of future energy request; and 3) prediction g_t of g.

3. ALGORITHM DESIGN

The key contribution of the work summarized here is the following decentralized algorithm that solves the ODLC problem in real-time. The key idea is to introduce a pseudo deferrable load q, simulated at the utility, to represent the power consumption of future deferrable loads. More specifically, solve the following problem at every time step t.

$$\begin{aligned}
\textbf{ODLC-t:}\quad \min\quad & \sum_{\tau=t}^{T} e^2(\tau) \\
\text{over}\quad & d_n(\tau), q(\tau), e(\tau), \quad n \leq N(t), \tau \geq t \\
\text{s.t.}\quad & e(\tau) = \sum_{n=1}^{N(t)} d_n(\tau) + q(\tau) - g_t(\tau), \quad \tau \geq t; \\
& \underline{d}_n(\tau) \leq d_n(\tau) \leq \overline{d}_n(\tau), \quad n \leq N(t),\ \tau \geq t; \\
& \sum_{\tau=t}^{T} d_n(\tau) = D_n(t), \quad n \leq N(t); \\
& \underline{q}(\tau) \leq q(\tau) \leq \overline{q}(\tau), \quad \tau \geq t; \\
& \sum_{\tau=t}^{T} q(\tau) = \mathbb{E}(A(t)),
\end{aligned}$$

where $D_n(t) = D_n - \sum_{\tau=1}^{t-1} d_n(\tau)$ is the energy to be consumed at or after time t, for all n and all t, and $\underline{q}$, $\overline{q}$ are given constants (from historical data) with $\underline{q}(t) = \overline{q}(t) = 0$.

Algorithm 1 solves ODLC-t at every time step t. Specifically, let $\mathcal{O}(t)$ denote the set of optimal schedules (d^*, q^*) to ODLC-t, and define $\text{dist}(d, \mathcal{O}(t)) := \min_{(d^*,q^*)\in\mathcal{O}(t)} \|d - d^*\|$ as the distance from d to $\mathcal{O}(t)$ for $t = 1, \ldots, T$.

Theorem 1. *At time $t = 1, \ldots, T$, the deferrable load schedules $d^{(k)}$ obtained by Algorithm 1 converge to optimal schedules of ODLC-t, i.e.,* $\text{dist}(d^{(k)}, \mathcal{O}(t)) \to 0$ *as* $k \to \infty$.

There is a simple characterization of the solutions of ODLC-t. Specifically, at time $t = 1, \ldots, T$, a feasible schedule (d, q) is called *t-valley-filling*, if there exists $C(t) \in \mathbb{R}$ such that

$$\sum_{n=1}^{N(t)} d_n(\tau) + q(\tau) - g_t(\tau) = C(t), \qquad \tau = t, \ldots, T. \quad (3)$$

Theorem 2. *At time $t = 1, \ldots, T$, a t-valley-filling deferrable load schedule, if exists, solves ODLC-t. Furthermore, in such cases, all optimal schedules to ODLC-t have the same aggregate load.*

This characterization serves as the basis for the performance analysis. Note that t-valley-filling schedules tend to exist if there is a large number of deferrable loads.

4. PERFORMANCE EVALUATION

In this section, we study the impact of the uncertainties about generation and deferrable load arrival, on the tracking error achieved by Algorithm 1; as well as the improvement of Algorithm 1 over the optimal open-loop control.

A generation prediction model is necessary to answer these two questions. In this paper, generation g is modeled as a random deviation δg around its expectation $\bar{g}$ as illustrated in Figure 1. The process δg is modeled as a sequence of uncorrelated random variables $e(1), \ldots, e(T)$, each with mean 0 and variance σ^2, passing through a causal filter with impulse response f ($f(\tau) = 0$ for $\tau < 0$), i.e.,

$$\delta g(\tau) = \sum_{s=1}^{T} e(s) f(\tau - s), \qquad \tau = 1, \ldots, T.$$

The prediction g_t at time $t = 1, \ldots, T$ is assumed to be[1]

$$g_t(\tau) = \bar{g}(\tau) + \sum_{s=1}^{t} e(s) f(\tau - s), \qquad \tau = 1, \ldots, T.$$

Throughout, we assume that a t-valley-filling schedule ex-

Algorithm 1 Real-time decentralized load control

At time step $t = 1, \ldots, T$,

Input: the utility knows g_t and $N(t)$, each deferrable load n knows $D_n(t)$, $\overline{d}_n$ and $\underline{d}_n$. The number K of iterations.

Output: $d_n(t)$ for $n = 1, \ldots, N(t)$.

(i) Set $k \leftarrow 0$. Each deferrable load $n \leq N(t)$ initializes its schedule $\{d_n^{(0)}(\tau)\}_{\tau \geq t}$ as

$$d_n^{(0)}(\tau) \leftarrow \begin{cases} d_n^{(K)}(\tau) & \text{if } n \leq N(t-1) \\ 0 & \text{if } n > N(t-1) \end{cases}, \quad \tau \geq t$$

where $d_n^{(K)}$ is the schedule in iteration K of the previous time step $t-1$.

(ii) The utility solves

$$\begin{aligned}
\min\quad & \sum_{\tau=t}^{T} \left(\sum_{n=1}^{N(t)} d_n^{(k)}(\tau) + q(\tau) - g_t(\tau) \right)^2 \\
\text{over}\quad & q(t), \ldots, q(T) \\
\text{s.t.}\quad & \underline{q}(\tau) \leq q(\tau) \leq \overline{q}(\tau), \quad \tau \geq t; \\
& \sum_{\tau=t}^{T} q(\tau) = \mathbb{E}(A(t))
\end{aligned}$$

to obtain $\{q^{(k)}(\tau)\}_{\tau \geq t}$, calculates $p^{(k)}$ as

$$p^{(k)}(\tau) \leftarrow \frac{1}{N(t)} \left(\sum_{n=1}^{N(t)} d_n^{(k)}(\tau) + q^{(k)}(\tau) - g_t(\tau) \right)$$

for $\tau \geq t$, and broadcasts $p^{(k)}$ to deferrable loads.

(iii) Each deferrable load $n \leq N(t)$ solves

$$\begin{aligned}
\min\quad & \sum_{\tau=t}^{T} p^{(k)}(\tau) d_n(\tau) + \frac{1}{2} \left(d_n(\tau) - d_n^{(k)}(\tau) \right)^2 \\
\text{over}\quad & d_n(t), \ldots, d_n(T) \\
\text{s.t.}\quad & \underline{d}_n(\tau) \leq d_n(\tau) \leq \overline{d}_n(\tau), \quad \tau \geq t; \\
& \sum_{\tau=t}^{T} d_n(\tau) = D_n(t),
\end{aligned}$$

to obtain $d_n^{(k+1)}$, and reports $d_n^{(k+1)}$ to the utility.

(iv) Set $k \leftarrow k + 1$. If $k < K$, go to Step (ii).

(v) Deferrable load $n \leq N(t)$ sets $d_n(t) \leftarrow d_n^K(t)$ and $D_n(t+1) \leftarrow D_n(t) - d_n(t)$.

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[1] This prediction algorithm is a Wiener filter [13].

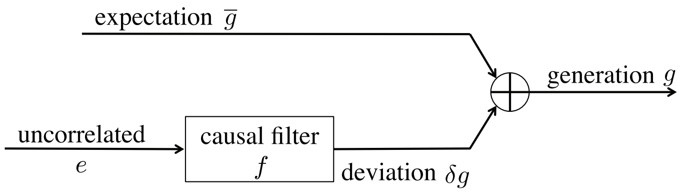

Figure 1: Diagram of the notation and structure of the model for generation.

ists at every time step t, under which one has

$$e(t) = \frac{1}{T-t+1}\left(\sum_{n=1}^{N(t)} D_n(t) + \mathbb{E}(A(t)) - \sum_{\tau=t}^{T} g_t(\tau)\right).$$

4.1 Expected mean square tracking error

The expected variance of tracking error

$$\mathbb{E}(V) = \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\left(e(t) - \frac{1}{T}\sum_{\tau=1}^{T} e(\tau)\right)^2\right]$$

achieved by Algorithm 1 is given in the following theorem.

Theorem 3. *The expected variance of tracking error* $\mathbb{E}(V)$ *obtained by Algorithm 1 is*

$$\mathbb{E}(V) = \frac{s^2}{T}\sum_{t=2}^{T}\frac{1}{t} + \frac{\sigma^2}{T^2}\sum_{t=0}^{T-1} F^2(t)\frac{T-t-1}{t+1} \quad (4)$$

where $F(t) = \sum_{s=0}^{t} f(s)$ *for* $t = 0, \ldots, T$.

Theorem 3 explicitly states how the generation prediction error (σ) and deferrable load prediction error (s) affect the expected mean square tracking error $\mathbb{E}(V)$. It follows immediately that $\mathbb{E}(V)$ tends to 0 as the predictions get precise, i.e., $\sigma \to 0$ and $s \to 0$.

Corollary 1. *The expected mean square tracking error* $\mathbb{E}(V) \to 0$ *as* $\sigma \to 0$ *and* $s \to 0$.

Besides, the impact of the time-correlation (given by f) in generation prediction error on $\mathbb{E}(V)$ is captured via F.

Corollary 2. *If* $|f(t)| \sim O(t^{-1/2-\alpha})$ *for some* $\alpha > 0$, *then* $\mathbb{E}(V) \to 0$ *as* $T \to \infty$.

4.2 Improvement over open-loop control

We compare the expected variance $\mathbb{E}(V)$ of tracking error obtained by Algorithm 1, with that (denoted $\mathbb{E}(V')$) obtained by the optimal open-loop control, which uses predictions at the beginning of the control window. Assume all deferrable loads arrive at time 1, i.e., $N(1) = N$, since otherwise open-loop control cannot obtain a feasible schedule for all deferrable loads.

The expected variance $\mathbb{E}(V')$ of tracking error obtained by the optimal open-loop control is given below.

Lemma 1. *If* $N(1) = N$, *then the expected variance* $\mathbb{E}(V')$ *of tracking error obtained by the optimal open-loop control is*

$$\mathbb{E}(V') = \frac{\sigma^2}{T^2}\sum_{t=0}^{T-1}\left(T(T-t)f^2(t) - F^2(t)\right).$$

Comparing $\mathbb{E}(V)$ and $\mathbb{E}(V')$ shows that Algorithm 1 always obtains a smaller expected tracking error than the optimal open-loop control. Specifically,

Corollary 3. *If* $N(1) = N$, *then*

$$\mathbb{E}(V') - \mathbb{E}(V) = \frac{\sigma^2}{T}\sum_{t=1}^{T}\frac{1}{2t}\sum_{m=0}^{t-1}\sum_{n=0}^{t-1}(f(m) - f(n))^2 \geq 0.$$

Corollary 3 highlights that Algorithm 1 is guaranteed to obtain a smaller expected tracking error than the optimal open-loop control. The next step is to quantify how much smaller $\mathbb{E}(V)$ is in comparison with $\mathbb{E}(V')$.

To do this we compute the ratio $\mathbb{E}(V')/\mathbb{E}(V)$ for two representative impulse responses $f(t)$, and the results are summarized in the following two corollaries.

Corollary 4. *If* $N(1) = N$ *and*

$$f(t) = \begin{cases} 1 & \text{if } 0 \leq t < \Delta \\ 0 & \text{otherwise,} \end{cases}$$

for some $\Delta > 0$, *then*

$$\frac{\mathbb{E}(V')}{\mathbb{E}(V)} = \frac{T/\Delta}{\ln(T/\Delta)}\left(1 + O\left(\frac{1}{\ln(T/\Delta)}\right)\right).$$

Corollary 5. *If* $N(1) = N$ *and* $f(t) = a^t$ *for some* $a \in (0, 1)$, *then*

$$\frac{\mathbb{E}(V')}{\mathbb{E}(V)} = \frac{1-a}{1+a}\frac{T}{\ln T}\left(1 + O\left(\frac{\ln\ln T}{\ln T}\right)\right).$$

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