

# Inefficiency in Forward Markets with Supply Friction

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**Abstract**—The growth of renewable resources will introduce significant variability and uncertainty into the grid. It is likely that “peaker” plants will be a crucial dispatchable resource for compensating for the variations in renewable supply. Thus, it is important to understand the strategic incentives of peaker plants and their potential for exploiting market power due to having responsive supply. To this end, we study an oligopolistic two-settlement market comprising of two types of generation (baseloads and peakers) where there is perfect foresight. We characterize symmetric equilibria in this context via closed-form expressions. However, we also show that, when the system is capacity-constrained, there may not exist equilibria in which baseloads and peakers play symmetric strategies. This happens because of opportunities for both types of generation to exploit market power to increase prices.

## I. INTRODUCTION

Concerns for global warming have led to greater emphasis on the development and deployment of renewable energy. In particular, several states have enacted renewable portfolio standards that call for utilities to expand their renewable portfolios. For example, California has set the goal that 33% of its electricity should come from renewable sources by 2020. Wind and solar energy are expected to play major roles in achieving these goals.

However, there are significant challenges in integrating wind and solar energy into current electric systems due to the intermittency and unpredictability of these energy sources [1]. The output from a single wind power plant can decrease from full output to zero in approximately 15 minutes. While wind power forecasts 1-2 hours ahead can achieve an accuracy level of approximately 4-12% (relative to installed wind capacity), the error could increase to 12-25% for day-ahead forecasts. Though techniques such as demand response and storage have been proposed as resources to balance fluctuations, these techniques are not yet widely adopted. Thus, for the foreseeable future, it seems that conventional dispatchable power plants will play an important role in the integration of renewable energy.

Dispatchable power plants are typically classified into two broad types “baseloads” and “peakers” [2], [3], [4]. The term “baseload” refers to generators that are suited for supplying base demand, i.e., they have long start-up times, slow ramping rates, and low marginal costs. For example, generation from nuclear (\$11.6/MWh) and coal plants (\$28.6/MWh) fall into the category of baseloads. In contrast, the term “peaker” refers to generators that are suited for supplying peak demand, i.e., they have short start-up times, fast ramping rates, and high marginal costs. For example, generation from gas turbine plants (\$48.0-\$79.9/MWh) fall into the category of peakers.

Traditionally, baseloads have provided the bulk of dispatchable load in electric systems since uncertainty has been fairly small and primarily on the demand side. However, in the future, it is expected that peakers will play a crucial role in managing the increasing supply-side uncertainty associated with the growth of intermittent, renewable generation [5], [6], [7]. A wind integration study conducted by a utility company found that a ten-minute market (as opposed to an hour-ahead market) could reduce wind integration costs by up to 60% per year by increasing incentives for flexible peaking resources [8].

### A. Contribution of this paper

Our focus in this paper is on the interaction between baseloads and peakers in electricity markets. In particular, we study the impact of generation responsiveness on strategic behavior in a forward market.

Due to the non-storable nature of electricity, electricity markets function in multiple stages, i.e., time-scales. In particular, “forward markets” ensure sufficient supply to meet the demand forecast while “spot markets” (a.k.a., “real-time markets”) ensure the instantaneous match of generation and demand. Thus, forward markets ensure that there is sufficient power procured ahead of time so that the probability of blackouts are low, and spot markets provide the opportunity to correct for errors in the forward forecasts.

In this paper, we study the behavior of baseloads and peakers within a two-settlement market, motivated by the structure of the electricity markets. In particular, we consider a setting with  $N_a$  symmetric peakers and  $N_b$  symmetric baseloads, where the differences between the two types of generators are captured by the following features: (i) peakers are capacity-constrained while baseloads are not, (ii) peakers have higher marginal cost than baseloads, and (iii) peakers have more production flexibility, i.e., baseloads can choose their production quantity only in the forward market while peakers can choose their production quantity in the spot market (and sell generation in both markets).

Within this context, we seek to understand the impact of the interaction between peakers, baseloads, and forward markets. In particular, we focus on the question of whether peakers can derive market power from their production flexibility or whether baseloads can derive market power from exploiting the capacity constraints of peakers. In general, we seek to understand when the strategic incentives of peakers and baseloads lead to inefficient market outcomes.

Our model, though simple, is already technically challenging to analyze. However, it yields novel insights into the questions above. We characterize the symmetric equilibria of the market via closed-form expressions. Our results reveal that there might not exist symmetric equilibria when the system is operating close to capacity. This happens because of opportunities for both types of generation to exploit market power to increase profits. Moreover, the two types of generation exploit market power through different

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mechanisms. Peakers reduce their forward positions and subsequently their productions in the spot market. On the other hand, baseloads will withhold productions ahead of time in the anticipation that their actions can drive up prices in the spot market. This is because, baseloads, who are less responsive than peakers, have incentives not to “over-produce” as they will not be able to adjust their productions downward as quickly should they desire to.

Our model, though simple, is already rich enough to highlight subtleties in the effects of capacity constraints. Our findings reveal that capacity constraints have non-trivial impacts and can create significant market power challenges (which would manifest as price fluctuations). These highlight the need to do market power analysis in order to understand if capacity constraints are creating a place where strategic incentives lead to poor market behavior. Since capacity constraints may be due to ramping issues, both long-term and operational market power analysis are important.

### B. Related literature

Related previous research includes work in oligopolistic forward markets and supply friction. We discuss each of these below.

*Forward Markets:* The earliest work on equilibria analysis of oligopolistic forward markets is a seminal paper by Allaz and Villa that demonstrated that forward markets can mitigate market power [9]. Their model was a two-stage Cournot game in which players take positions in the forward market in the first stage and choose productions in the second stage. Subsequently, there has been numerous work on reaffirming or invalidating the results by Allaz and Villa under more relaxed assumptions, e.g. [10], [11], [12], [13]. Recently in [14], [15], the Allaz Villa model was extended to incorporate network constraints and price caps – important features of electricity markets. However, due to the complexity of the resulting problem, only numerical solutions were provided. In [16], the authors argued that forward trading could induce long-term capacity investment, which sparked research into the relationship between forward contracts and capacity investment, e.g. [17], [18].

*Supply friction:* Motivated by the events in the California wholesale electricity market between 1998-2000, the authors in [19] very recently proposed and analyzed a dynamic general equilibrium model with supply friction (a term they introduced to refer to constraints on the production of commodity). They discovered that, even though the competitive equilibrium is efficient, the market price is highly volatile. In particular, the market price fluctuates between zero and a “choke-up” price, without any tendency to converge to the marginal production cost. And even though players are price-takers, the surplus from trading is skewed in favor of suppliers. Subsequent work extended the model in [19] to study the impact of transmission constraints [20] and increased wind penetration [21]. The authors find that transmission constraints can exacerbate the price volatility, and that there exist thresholds for the coefficient of variation beyond which the value of wind is questionable. To our knowledge, there is no prior work on the impact of supply friction in strategic settings, which is the focus of the current paper.

## II. MODEL

Our goal in this paper is to understand the strategic incentives of peaker plants and their potential for exploiting

market power due to possessing more responsive supply. Peaker plants have more flexibility to adjust production quantities than baseload plants. Hence, a key feature we seek to capture in our model is how the flexibilities of peaker plants and baseload plants affect their participation in the markets.

*Throughout, we focus on the markets for delivery of electricity at a single instant of time.*

### A. Two-settlement market

Our two-settlement market model is motivated by the predominant literature on forward markets [9], [10], [11], [12], [13], [14], [15]. Electricity is typically traded in multiple markets prior to delivery. In our model, we assume that electricity is traded in two stages prior to delivery. We refer to the first stage as the forward market and the second stage as the spot market. In the forward market, generators sign contracts with retailers to deliver a certain quantity of electricity at a price  $p_f$ . We assume that these contracts are binding and observable precommitments. In the spot market, generators sell electricity to retailers at a price  $P(q)$  which is a function of the total quantity  $q$  of electricity purchased by retailers in both the forward and spot markets. More specifically, we assume a linear demand function:

$$P(q) = \xi - \frac{b}{N}q$$

where  $b$  and  $N$  are constant parameters. Note that this assumption is common, e.g., [9], [18], and corresponds to modeling retailers as having a quadratic revenue function.

Motivated by the work in [9], we assume there is perfect foresight. Hence, the forward price and the spot price are aligned, i.e.,  $p_f = P(q)$ . An extension to the case of uncertain demand is, of course, relevant and interesting. But, the case of certain demand is challenging and interesting enough that uncertain demand is left for future work.

We have assumed that there are no network constraints. This is a common simplification in prior studies of market power [9], [10], [11], [12], [13], [18]. The case with network constraints is certainly relevant. However, it is reasonable to ignore underlying network constraints in this paper given that our focus is on market power that arises from ramp constraints. This assumption also makes the model tractable, allowing for detailed analysis and stronger characterizations of market equilibria.

### B. Generation types

As described earlier, we consider only two types of generation in this paper. We refer to the more responsive generation as peakers and the less responsive generation as baseloads. We assume that there are  $N_a$  peakers and  $N_b$  baseloads. All peakers are identical and all baseloads are identical. Peakers have constant marginal costs  $c_a > 0$ , while baseloads have constant marginal costs  $c_b > 0$ , where  $c_a \geq c_b$ .

We index peakers by  $i$  and baseloads by  $j$ , hence,  $i = 1, \dots, N_a$  and  $j = 1, \dots, N_b$ . Let  $q_{a,i}$  and  $q_{b,j}$  denote the quantities produced by peaker  $i$  and baseload  $j$  respectively. To model baseloads having to decide on production quantities ahead of peakers, we assume that  $q_{a,i}$  is chosen in the spot market while  $q_{b,j}$  is chosen in the forward market. Let  $q_{f,i}$  denote the quantity sold by peaker  $i$  in the forward market. We assume that baseloads sell all their productions in the forward market (this is possible since baseloads choose

productions in the forward market). Hence, baseload  $j$  sells the quantity  $q_{b,j}$  in the forward market.

We assume that each peaker has a production capacity  $k_a > 0$  and that baseloads are not capacity constrained, i.e.,  $0 \leq q_{a,i} \leq k_a$  for all  $i$  and  $q_{b,j} \geq 0$  for all  $j$ . The capacity constraints on peakers can be interpreted as ramp constraints of generators. In practice, peakers would only be able to adjust their productions within a limited range about their existing operating points. Hence, a more sophisticated model would have peakers choose set points in the forward market, and impose constraints on the allowed adjustment from the set points. In this paper, we adopt the simplified model where peakers have a set point of zero and can ramp up to a maximum of  $k_a$ . Similarly, the constraint that baseloads choose productions in the forward market can be interpreted as baseloads choosing operating points in the forward market and not being allowed to deviate from the set point in the spot market.

### C. Competitive model

Given the market structure (forward and spot) and the market players (baseload and peakers), we can now describe the solution concept we adopt for the market. In particular, our market equilibria is defined as follows.

Let the vectors  $\mathbf{q}_f = (q_{f,1}, \dots, q_{f,N_a})$ ,  $\mathbf{q}_b = (q_{b,1}, \dots, q_{b,N_b})$ , and  $\mathbf{q}_a = (q_{a,1}, \dots, q_{a,N_a})$  denote the forward quantities sold by the peakers, the quantities sold by the baseloads, and the total quantities sold by the peakers, respectively. We also use the notation  $\mathbf{q}_{f,-i} = (q_{f,1}, \dots, q_{f,i-1}, q_{f,i+1}, \dots, q_{f,N_a})$  to denote the vector of forward quantities sold by all peakers other than  $i$ . Similarly, we use  $\mathbf{q}_{b,-j} = (q_{b,1}, \dots, q_{b,j-1}, q_{b,j+1}, \dots, q_{b,N_b})$  and  $\mathbf{q}_{a,-i} = (q_{a,1}, \dots, q_{a,i-1}, q_{a,i+1}, \dots, q_{a,N_a})$ .

*Spot market (Peaker):* We start by defining the spot market equilibrium. Note that only peakers can play in the spot market. Peaker  $i$ 's profit from the spot market is given by:

$$\begin{aligned} \pi_{a,i}^{(s)}(q_{a,i}; \mathbf{q}_{a,-i}) \\ = P \left( \sum_{i'} q_{a,i'} + \sum_{j'} q_{b,j'} \right) (q_{a,i} - q_{f,i}) - c_a q_{a,i}. \end{aligned}$$

Given  $\mathbf{q}_{a,-i}$ , each peaker  $i$  chooses  $q_{a,i}$  to maximize its profit  $\pi_{a,i}^{(s)}(q_{a,i}; \mathbf{q}_{a,-i})$  subject to the constraint that  $0 \leq q_{a,i} \leq k_a$ . A Nash equilibrium of the spot market game defined by  $(\pi_{a,1}^{(s)}, \dots, \pi_{a,N_a}^{(s)})$  is a vector  $\mathbf{q}_a$  such that for all  $i$ :

$$\pi_{a,i}^{(s)}(q_{a,i}; \mathbf{q}_{a,-i}) \geq \pi_{a,i}^{(s)}(\bar{q}_{a,i}; \mathbf{q}_{a,-i}), \text{ for all } \bar{q}_{a,i} \in [0, k_a].$$

Let  $\mathbf{q}_a^*(\mathbf{q}_f, \mathbf{q}_b) = (q_{a,1}^*(\mathbf{q}_f, \mathbf{q}_b), \dots, q_{a,N_a}^*(\mathbf{q}_f, \mathbf{q}_b))$  denote a spot market equilibrium. We will address the issue of whether  $\mathbf{q}_a^*(\mathbf{q}_f, \mathbf{q}_b)$  is well-defined after we describe the forward game. For notational brevity, we will often omit the dependence of  $\mathbf{q}_a^*$  on  $\mathbf{q}_f$  and  $\mathbf{q}_b$ .

*Forward market:* The forward market equilibrium depends on both the peaker and baseload behavior. First, note that the strategy of peakers in the forward market depends on the outcome of the spot market. In particular, Peaker  $i$ 's profit

from the forward market is given by:

$$\begin{aligned} \pi_{a,i}^{(f)}(q_{f,i}; \mathbf{q}_{f,-i}, \mathbf{q}_b) \\ = P \left( \sum_{i'} q_{a,i'}^* + \sum_{j'} q_{b,j'} \right) q_{f,i} + \pi_{a,i}^{(s)}(q_{a,i}^*; \mathbf{q}_{a,-i}) \\ = \left( P \left( \sum_{i'} q_{a,i'}^* + \sum_{j'} q_{b,j'} \right) - c_a \right) q_{a,i}^* \end{aligned}$$

where the second equality follows by substituting for  $\pi_{a,i}^{(s)}(q_{a,i}^*; \mathbf{q}_{a,-i})$ . Given  $\mathbf{q}_{f,-i}$  and  $\mathbf{q}_b$ , each peaker  $i$  chooses  $q_{f,i}$  to maximize its profit  $\pi_{a,i}^{(f)}(q_{f,i}; \mathbf{q}_{f,-i}, \mathbf{q}_b)$ . This is an unconstrained maximization as peakers can take positive or negative positions in the forward market.

Next, we can write Baseload  $j$ 's profit as:

$$\pi_{b,j}^{(f)}(q_{b,j}; \mathbf{q}_{b,-j}, \mathbf{q}_f) = \left( P \left( \sum_{i'} q_{a,i'}^* + \sum_{j'} q_{b,j'} \right) - c_b \right) q_{b,j}.$$

Given  $\mathbf{q}_{b,-j}$  and  $\mathbf{q}_f$ , each baseload  $j$  chooses  $q_{b,j}$  to maximize its profit  $\pi_{b,j}^{(f)}(q_{b,j}; \mathbf{q}_{b,-j}, \mathbf{q}_f)$  subject to the constraint that  $q_{b,j} \geq 0$ .

Combining the peaker and baseload profit equations, a Nash equilibrium of the forward market game defined by  $(\pi_{a,1}^{(f)}, \dots, \pi_{a,N_a}^{(f)}, \pi_{b,1}^{(f)}, \dots, \pi_{b,N_b}^{(f)})$  is a tuple  $(\mathbf{q}_b, \mathbf{q}_f)$  such that for all  $i$ :

$$\pi_{a,i}^{(f)}(q_{f,i}; \mathbf{q}_{f,-i}, \mathbf{q}_b) \geq \pi_{a,i}^{(f)}(\bar{q}_{f,i}; \mathbf{q}_{f,-i}, \mathbf{q}_b), \text{ for all } \bar{q}_{f,i} \in \mathbf{R}$$

and for all  $j$ :

$$\pi_{b,j}^{(f)}(q_{b,j}; \mathbf{q}_{b,-j}, \mathbf{q}_f) \geq \pi_{b,j}^{(f)}(\bar{q}_{b,j}; \mathbf{q}_{b,-j}, \mathbf{q}_f), \text{ for all } \bar{q}_{b,j} \geq 0.$$

It is this equilibrium that is the focus of the remainder of the paper. As the above definitions make clear, even though the model is simplified, the market equilibria are already quite complex and technically challenging. There are certainly important features that are excluded from the model, e.g., uncertainty in demand and network constraints, but already the model is rich enough to shed light on the interaction between forward markets and generation flexibility.

Because it is challenging to perform a general analyses of the forward equilibria, in this paper, we focus on equilibria in which peakers have symmetric forward positions and baseloads have symmetric productions. Despite this restriction, our results reveal interesting interactions between peakers and baseloads. We also restrict ourselves to spot market Nash equilibria in which peakers that have equal forward positions necessarily have equal spot productions. These restrictions ensure that  $\mathbf{q}_a^*(\mathbf{q}_f, \mathbf{q}_b)$  is well-defined for our purpose.<sup>1</sup>

- 1) In Section III, we fix the actions of the baseloads symmetrically and solve for symmetric equilibria among peakers.

<sup>1</sup>Due to our restriction, in our analysis of the forward market, we need only consider strategies in which a single producer (peaker/baseload) deviated. It turns out that, the forward strategies that are relevant to our analysis will always induce a subgame (spot market) that has a unique Nash equilibrium (within our restriction on the spot market). Hence, for the purposes of this paper,  $\mathbf{q}_a^*(\mathbf{q}_f, \mathbf{q}_b)$  is well-defined.

- 2) In Section IV, we fix the actions of the peakers symmetrically and solve for symmetric equilibria among baseloads.
- 3) In Section V, we use the “best-response” functions obtained in steps 1 and 2 to solve for equilibria of the forward market.

### III. PEAKER EQUILIBRIUM

In this section, we fix the actions of the baseloads  $\mathbf{q}_b = q_b \mathbf{1}$  (for some  $q_b \geq 0$ ) in the forward market and solve for the symmetric best response  $q_f \mathbf{1}$  of the peakers. That is to say, for all  $i$ :

$$\pi_{a,i}^{(f)}(q_f; q_f \mathbf{1}, q_b \mathbf{1}) \geq \pi_{a,i}^{(f)}(\bar{q}_{f,i}; q_f \mathbf{1}, q_b \mathbf{1}), \text{ for all } \bar{q}_{f,i} \in \mathbf{R}.$$

Let  $Q_f^*(q_b) \subset \mathbf{R}$  denote the set of all symmetric peaker responses  $q_f$  (there could be multiple peaker responses). It turns out that, for a given baseload production  $q_b$ , all the peaker responses will lead to the same symmetric peaker production in the spot market. We denote that symmetric peaker production by  $q_a^*(q_b)$ .

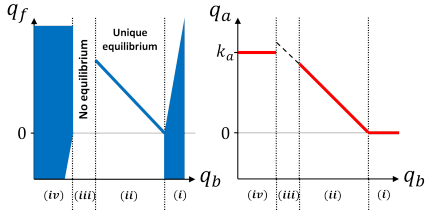


Fig. 1: Symmetric peaker response set  $Q_f^*(q_b)$  and the subgame equilibrium peaker production  $q_a^*(q_b)$ .

Fig. 1 shows representative plots of  $Q_f^*(q_b)$  and  $q_a^*(q_b)$ . There are four characteristic segments labelled (i)–(iv). In general, we expect the peakers’ reactions to decrease as  $q_b$  increases because a higher baseload production decreases the effective demand in the spot market. This behavior indeed holds in segment (ii) where capacity constraints are never binding. However, the behavior in the rest of the segments are significantly different. In segments (i) and (iv), there are infinitely many Nash equilibria characterized by half-lines. These are degenerate corner cases where peakers are neutral to a range of forward positions that lead to the same outcome in the spot market.

In segment (iii), there does not exist any Nash equilibrium among the peakers. This is due to peakers exploiting market power in the forward market when their productions approach capacity. When a peaker  $i$  reduces its forward position  $q_{f,i}$ , it induces peaker  $-i$  to increase its spot production  $q_{a,-i}$ . But peaker  $-i$  cannot increase its production beyond its capacity. Hence, the total spot production decreases, the market price increases, and peaker  $i$ ’s profits increase. By a similar argument, peaker  $-i$  has an incentive to reduce its forward position to increase its profits. Yet, should all peakers choose to reduce their forward positions, there will be excess demand in the market, which would drive all peakers to increase their forward positions. Hence, there is no symmetric Nash equilibrium between the peakers.

*Formal statement:* We state our results for  $Q_f^*(q_b)$  formally in Proposition 1.<sup>2</sup>

<sup>2</sup>Due to lack of space, we omit all proofs from our paper and refer the reader to [22] for the proofs.

**Proposition 1.** Let  $\xi_a = (N/b)(\xi - c_a)$ . Suppose  $N_a > 1$ . Suppose that baseloads have symmetric productions  $\mathbf{q}_b = q_b \mathbf{1}$  for some  $q_b \geq 0$ . Let  $Q_f^*(q_b) \subset \mathbf{R}$  denote the set of all symmetric peaker reactions, i.e. for all  $q_f \in Q_f^*(q_b)$ , the vector of forward positions  $q_f \mathbf{1}$  is a Nash equilibrium of the game between the peakers. Then there exists constants  $q_b^{(1)}$ ,  $q_b^{(2)}$ ,  $q_b^{(3)}$  such that:

$$Q_f^*(q_b) = \begin{cases} [-\xi_a + N_b q_b + (N_a + 1)k_a, \infty), & \text{if } q_b \leq q_b^{(3)}; \\ \emptyset, & \text{if } q_b^{(3)} < q_b < q_b^{(2)}; \\ \left\{ \frac{N_a - 1}{N_a^2 + 1} (\xi_a - N_b q_b) \right\}, & \text{if } q_b^{(2)} \leq q_b \leq q_b^{(1)}; \\ (-\infty, -\xi_a + N_b q_b], & \text{if } q_b^{(1)} < q_b. \end{cases}$$

Moreover, for any symmetric peaker reaction  $q_f \in Q_f^*(q_b)$ , the equilibrium spot productions are given by:

$$\mathbf{q}_a^*(q_f \mathbf{1}, q_b \mathbf{1}) = \begin{cases} k_a \mathbf{1}, & \text{if } q_b \leq q_b^{(3)}; \\ \frac{N_a}{N_a^2 + 1} (\xi_a - N_b q_b) \mathbf{1}, & \text{if } q_b^{(2)} \leq q_b \leq q_b^{(1)}; \\ \mathbf{0}, & \text{if } q_b^{(1)} < q_b. \end{cases}$$

### IV. BASELOAD EQUILIBRIUM

Next, we fix the actions of the peakers  $\mathbf{q}_f = q_f \mathbf{1}$  (for some  $q_f \in \mathbf{R}$ ) in the forward market and solve for the symmetric best response of the baseloads  $q_b \mathbf{1}$ . That is to say, for all  $j$ :

$$\pi_{b,j}^{(f)}(q_b; q_b \mathbf{1}, q_f \mathbf{1}) \geq \pi_{b,j}^{(f)}(\bar{q}_{b,j}; q_b \mathbf{1}, q_f \mathbf{1}), \text{ for all } \bar{q}_{b,j} \geq 0.$$

Let  $Q_b^*(q_f) \subset \mathbf{R}$  denote the set of all symmetric baseload responses  $q_b$  (there could be multiple baseload responses).

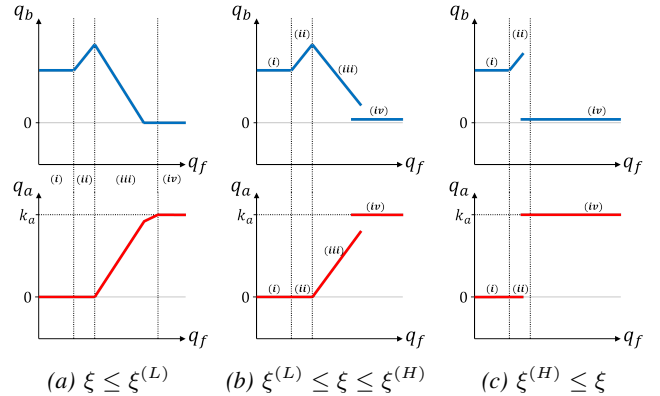


Fig. 2: Symmetric baseload response set  $Q_b^*(q_f)$  and the subgame equilibrium peaker production  $q_a^*(q_f \mathbf{1}, q_b \mathbf{1})$ .

Fig. 2 shows representative plots of  $Q_b^*(q_f)$ . These solutions are differentiated by the level of demand  $\xi$  with demand being highest in scenario (c). Moreover, the transition from scenarios (a) to (b) has an intuitive interpretation: in scenario (b), demand is high enough that baseloads have incentive to produce even when all peakers are producing at capacity  $k_a$ . To understand this, note that the constant  $\xi^{(L)}$  is given by:

$$\xi^{(L)} = c_b + \frac{b}{N} N_a k_a.$$

Suppose all peakers are producing at capacity. Then the baseload’s profit margin on each unit of electricity is given by  $\xi - (b/N)(N_a k_a + \sum_j q_{b,j}) - c_b$ . In case (a), this profit

margin is always negative. Hence, the shape of the reaction curves for scenarios (a) and (b) differ only in segment (iv) where peakers' productions are at capacity  $k_a$ .

In general, we expect the baseload reaction to decrease as  $q_f$  increases because larger forward positions lead to larger spot productions which in turn exert a downward pressure on the market price. This behavior indeed holds in segment (iii). However, the behavior in the rest of the segments is significantly different. In segment (ii), the baseload reaction increases as  $q_f$  increases, and in segments (i) and (iv), the baseload reaction is constant. Segments (i) and (iv) are degenerate corner cases in which peakers produce either zero or  $k_a$  regardless of their forward positions and hence the baseload also produces a constant quantity.

Segment (ii) can be attributed to the baseload exercising its first-mover advantage to drive the peaker production to zero when peakers increase their participation in the market. By increasing their commitments to produce, baseloads are able to drive peakers out of the spot market. Hence peakers still produce zero even though their forward positions have increased.

The most important feature is that the reaction curve is *discontinuous* between segments (iii) and (iv). This phenomena is attributed to baseload withholding production when peakers are operating near capacity (even though doing so would drive peakers to increase their spot productions). Because peakers are capacity constrained, the increase in peakers' spot productions is less than the amount of production that the baseloads withheld. Hence, the market price increases, and the baseloads' profits increase.

*Formal statement:* We now state our results for  $Q_b^*(q_f)$  formally in Proposition 2. However, we encourage the reader to refer to the figures and preceding discussion for the important structural features.

**Proposition 2.** Let  $\xi_b = (N/b)(\xi - c_b)$  and  $c_m = (N/b)(c_a - c_b)$ . Suppose that peakers have symmetric forward positions  $\mathbf{q}_f = q_f \mathbf{1}$  for some  $q_f \in \mathbf{R}$ . Let  $Q_b^*(q_f) \subset \mathbf{R}$  denote the set of all symmetric baseload reactions, i.e. for all  $q_b \in Q_b^*(q_f)$ , the vector of productions  $q_b \mathbf{1}$  is a Nash equilibrium of the game between the baseloads. Then  $Q_b^*(q_f)$  is the union of two sets that each contain at most one element:

$$Q_b^*(q_f) = Q_b'(q_f) \cup Q_b''(q_f),$$

where  $Q_b'(q_f)$  and  $Q_b''(q_f)$  are defined by:

$$Q_b'(q_f) = \begin{cases} \left\{ \frac{1}{N_b+1} \xi_b \right\}, & \text{if } q_f \leq q_f^{(i)}; \\ \left\{ \frac{1}{N_b} (\xi_b - c_m + q_f) \right\}, & \text{if } q_f^{(i)} < q_f \leq q_f^{(ii)}; \\ \left\{ \frac{1}{N_b+1} (\xi_b + N_a (c_m - q_f))^+ \right\}, & \text{if } q_f^{(ii)} < q_f \leq q_f^{(iii)}; \\ \emptyset, & \text{if } q_f^{(iii)} < q_f, \end{cases}$$

$$Q_b''(q_f) = \begin{cases} \emptyset, & \text{if } q_f < q_f^{(iv)}; \\ \left\{ \frac{1}{N_b+1} (\xi_b - N_a k_a)^+ \right\}, & \text{if } q_f^{(iv)} \leq q_f. \end{cases}$$

for some constants  $q_f^{(i)}$ ,  $q_f^{(ii)}$ ,  $q_f^{(iii)}$ ,  $q_f^{(iv)}$  satisfying  $q_f^{(i)} \leq q_f^{(ii)} \leq q_f^{(iii)}$ . Moreover, the following statements hold:

- (a)  $\mathbf{q}_a^*(q_f \mathbf{1}, q_b \mathbf{1}) = \mathbf{0}$  for all  $q_f \leq q_f^{(ii)}$  and  $q_b \in Q_b'(q_f)$ .
- (b)  $\mathbf{q}_a^*(q_f \mathbf{1}, q_b \mathbf{1}) = k_a \mathbf{1}$  for all  $q_f \geq q_f^{(iv)}$  and  $q_b \in Q_b''(q_f)$ .

## V. MARKET EQUILIBRIUM

Next, we solve for symmetric Nash equilibria of the forward market. Our analysis involves solving for the points of intersection of the peaker reaction curves in Fig. 1 and the baseload reaction curves in Fig. 2. Unfortunately, there are many ways in which those curves could intersect and an exhaustive characterization would digress from the important features. As a result, in this paper, we consider only the scenario where  $c_a = c_b$ , and focus on four possible cases which are illustrated in Fig. 3.

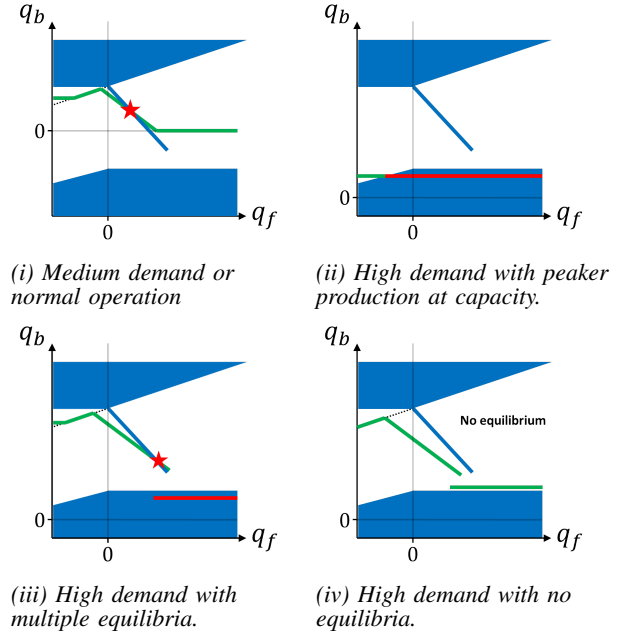


Fig. 3: Selection of four possible symmetric forward equilibrium  $(q_b \mathbf{1}, q_f \mathbf{1})$ . Peaker equilibria curves are plotted in blue and baseload equilibria curves are plotted in green. Their intersection (i.e. the forward equilibria) are marked in red.

Case (i) is a case of medium demand where production constraints are not binding. Case (ii) is a case of high demand in which peakers are operating at capacity but there are no observable adverse behavior. Case (iii) and (iv) illustrate two adverse behavior that could arise when demand is high and peakers are operating near capacity. These adverse behavior are due to baseloads and peakers having market power and incentives to withhold production. For notational convenience, we let  $q_a \mathbf{1}$  denote the unique symmetric peaker production in the subgame associated with each of the scenarios (i)–(iv).

(i) *Medium demand or normal operation:* Here,  $0 < q_a < k_a$ . In this case, both baseloads and peakers are producing in the market. Peakers are not capacity constrained. This case can be interpreted as having medium demand in the system.

(ii) *High demand with peaker production at capacity:* Here,  $q_a = k_a$ . Hence, peakers are capacity constrained. There are infinitely many equilibria in peakers' forward positions. This case can be interpreted as having very high demand in the system.

(iii) *High demand with multiple equilibria:* Here, two structurally different equilibria can be sustained – one with  $q_a = k_a$  and one with  $0 < q_a < k_a$ . This case can be interpreted as having high demand, but not high enough

to have peakers always operating at capacity. As illustrated in Fig. 3iii, there could be two equilibria in baseload productions corresponding to the same peaker forward position. This shows that, due to high demand, baseloads have market power and are able to profit by withholding production and having peakers operate at capacity.

(iv) *High demand with no equilibria:* Here, there are no equilibria. This case can be interpreted as having high demand in the system, but not high enough to have peakers operate at capacity. This phenomena is caused by both peakers and baseloads wanting to exploit market power. When demand is high and peakers are operating near capacity, there is no equilibria in peakers' forward positions (as shown in segment (iii) in Fig. 1). Baseloads are also able to profit by withholding production (as shown in the discontinuity between segments (iii) and (iv) in Fig. 2b and 2c).

*Formal statement:* We now state our results for the symmetric Nash equilibria of the forward market formally in Proposition 3 for the scenario where  $c_a = c_b$ . Our results for the case of general marginal costs are given in [22].

**Proposition 3.** *Suppose  $c_a = c_b = c$ . Let  $\xi_b = (N/b)(\xi - c)$ . Let  $Q \subset \mathbf{R}^2$  denote the set of all symmetric forward equilibria, i.e. for all  $(q_b, q_f) \in Q$ , the vector of forward positions  $q_f \mathbf{1}$  and the vector of baseload productions  $q_b \mathbf{1}$  is a Nash equilibrium of the forward market. Then:*

$$Q = Q' \cup Q'',$$

where  $Q'$  and  $Q''$  are defined by:

$$Q' = \left\{ \begin{array}{l} (q_b, q_f) : \left. \begin{array}{l} q_b = \frac{N_a+1}{N_a^2+N_a N_b+N_b+1} \xi_b, \\ q_f = \frac{N_a-1}{N_a^2+N_a N_b+N_b+1} \xi_b \end{array} \right\}, \text{ if } 0 < \xi_b \leq \xi_b^{(i)}; \\ \emptyset, \text{ if otherwise,} \end{array} \right.$$

$$Q'' = \left\{ \begin{array}{l} (q_b, q_f) : \left. \begin{array}{l} q_b = \frac{1}{N_b+1} (\xi_b - N_a k_a), \\ q_f \geq q_f^{(i)}(\xi_b) \end{array} \right\}, \text{ if } \xi_b^{(ii)} \leq \xi_b; \\ \emptyset, \text{ if otherwise,} \end{array} \right.$$

for some constants  $\xi_b^{(i)}$  and  $\xi_b^{(ii)}$  and some function  $q_f^{(i)}(\xi_b)$ .

## VI. CONCLUSION

Our goal in this paper is to study the impact of generation responsiveness on strategic behavior in forward markets for electricity. To this end, we consider a two-settlement market comprising two types of generators – peakers and baseloads – where the difference between the two types of generators is captured by having baseloads choose their production quantities in the forward market while peakers choose their production quantities in the spot market (and sell generation in both markets). We characterize the symmetric equilibria of this market via closed-form expressions. Our results reveal that there might not exist symmetric equilibria when the system is operating close to capacity. This happens because of opportunities for both types of generation to exploit market power to increase profits. Moreover, baseloads' constraints on having to commit to productions ahead of peakers gives baseloads a first-mover advantage over the peakers.

Hence, our findings reveal that capacity constraints have non-trivial impacts and can create significant market power challenges (which would manifest as price fluctuations). These highlight the need to do market power analysis to understand if capacity constraints create a place where strategic incentives lead to poor market behavior. Since capacity

constraints may be due to ramping issues, both long-term and operational market power analysis are important.

There are numerous future research directions that build on the work in this paper. An interesting direction would be to consider a simpler setup with fewer producers (e.g. one baseload and two peakers) and study the asymmetric equilibria and mixed equilibria. Another important direction would be to incorporate uncertainty in demand/supply. While we expect the structural findings of this work to continue to hold when demand uncertainty is low, it is unclear what the results would be if demand is highly variable. Another extension would be to model running market instances instead of focusing on a single instant of time as it raises the question of whether ramping constraints will continue to have an impact on strategic behavior over longer time scales.

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