Swing Dynamics as Primal-Dual Algorithm for Optimal Load Control

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***Abstract*—In electricity transmission networks, loads can provide flexible, fast responsive, and decentralized sources for frequency regulation and generation-demand balancing, complementary to generation control. We consider an *optimal load control (OLC)* problem in a transmission network, when a disturbance in generation occurs on an arbitrary subset of the buses. In *OLC*, the frequency-insensitive loads are reduced (or increased) in real-time in a way that balances the generation shortfall (or surplus), resynchronizes the bus frequencies, and minimizes the aggregate disutility of load control. We propose a frequency-based load control mechanism and show that the swing dynamics of the network, together with the proposed mechanism, act as a decentralized primal-dual algorithm to solve *OLC*. Simulation shows that the proposed mechanism can resynchronize the bus frequencies, balance demand with generation and achieve the optimum of *OLC* within several seconds after a disturbance in generation. Through simulation, we also compare the performance of the proposed mechanism with automatic generation control (AGC), and discuss the effect of their incorporation.**

I. Introduction

In power networks, the mismatch between generation and load may cause frequency deviations from its nominal value. Such frequency deviations, if not tightly controlled around zero, may bring instability to the system or even damage the facilities. Therefore, frequency regulation and generation-demand balancing are important.

Traditional regulation efforts rely on generation side. Automatic generation control (AGC) is a good example [1][2]. However, relying solely on generation may not be enough. Due to limited ramping rate, generators are suitable for minute-to-minute regulation, but may incur expensive wear-and-tear, high emissions and low thermal efficiency when responding to regulation signals in intervals of seconds [3][4]. Complementary to generators, loads provide low cost and fast responsive regulation sources.

Feasibility and efficiency of load control has been justified in several electricity markets. ERCOT has 50% of its 2400 MW reserve provided by loads. PJM opened up the regulation market to participation by loads [3]. While most of the installed programs focused on direct manipulation of loads in a centralized mechanism, an alternative strategy, decentralized load control via frequency measurement, has been studied broadly in literatures. Brooks *et al.* suggested that loads can respond to frequency and provide regulation within 1 second [4]. Molina-Garcia *et al.* studied the aggregate response characteristics when individual loads are turned on/off as the frequency goes across certain regions [5]. Donnelly *et al.* developed proportional frequency feedback control of intelligent loads, and investigated the effect of distribution systems, the effect of discretized control, and the effect of time-delay of control actions, using a 16-generator transmission network simulation test bed [6]. Literature review shows that frequency-based load control does not rely much on the communication to the centralized grid operator, thus suitable for large-scale, decentralized deployment [5][6].

Different from papers above, we provide a fresh and unified interpretation within an optimization framework for frequency-based load control. Consider a transmission network in steady state where the generator frequencies at different buses (or in different balancing authorities) are synchronized to the same nominal value and the mechanic power is balanced with the electric power at each bus. Suppose a small disturbance in generation occurs on an arbitrary subset of the buses. How should the frequency-insensitive, controllable loads in the network be reduced (or increased) in real time in a way that (i) balances the generation shortfall (or surplus), (ii) resynchronizes the bus frequencies, and (iii) minimizes a measure of aggregate disutility of participation in such a load control? We formalize these questions as an *optimal load control (OLC)* problem. Then, we develop a frequency-based load control mechanism where loads are controlled as the inversed marginal disutility function of locally measured frequency. As a result of reverse engineering, such a frequency-based load control coupled with the dynamics of swing equations and power flows serve as a distributed primal-dual algorithm to solve *OLC*. Simulation on a 16-generator test bed shows that the proposed mechanism can restore frequency, balance load with generation and achieve the optimum of *OLC* within several seconds after a disturbance in generation. Moreover, we compare the performance of the proposed mechanism with AGC, and show with simulation that adding the proposed mechanism can improve the transient performance of AGC.

The paper is organized as follows. Section II describes the dynamics of transmission networks. Section III introduces the *optimal load control (OLC)* problem. Section IV interprets how the frequency-based load control and swing dynamics serve as a primal-dual algorithm to solve *OLC*. Section V shows simulation-based case studies. Finally, Section VI provides concluding remarks.

II. Transmission network dynamics

The transmission network is described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \dots, N\}$ is the set of buses and $\mathcal{E}$ is the set of transmission lines connecting the buses. We adopt the following assumptions [1]

- The lines $(i,j) \in \mathcal{E}$ are lossless and characterized by their reactance $\mathbf{i}x_{ij}$.
- The voltage magnitudes $|V_j|$ of buses $j \in \mathcal{V}$ are constant.
- Reactive power injections at the buses and reactive power flows on the lines are ignored.

We assume that $\mathcal{G}$ is directed, with an arbitrary orientation, so that if $(i,j) \in \mathcal{E}$, then $(j,i) \notin \mathcal{E}$. We use (i,j) and $i \to j$ interchangeably to denote a link in $\mathcal{E}$. We also assume without loss of generality that $\mathcal{G}$ is connected. To simplify notation, we assume all variables represent deviations from their nominal (operating) values and are in per unit.

The dynamics at bus j with a generator is modeled by the swing equation

$$M_j \dot{\omega}_j = P_j^m - P_j^e, \tag{1}$$

where ω_j is the frequency deviation from its nominal value, M_j is the inertia constant of the generator, P_j^m is the deviation in mechanic power injection to bus j from its nominal value, and P_j^e is the deviation in electric power from its nominal value. Each bus may have two types of loads, *frequency-sensitive* (e.g. motor-type) loads and *frequency-insensitive* (but controllable) loads. The total change $\hat{d}_j$ in frequency-sensitive loads at bus j as a function of the frequency deviation ω_j is $\hat{d}_j := D_j \omega_j$, where D_j is the damping constant. Let $\mathcal{L}(j)$ denote the set of frequency-insensitive loads at bus j, and $(d_l, l \in \mathcal{L}(j))$ denote the deviations of frequency-insensitive loads from their nominal values. Then the electric power P_j^e is the sum of all frequency-sensitive loads, frequency-insensitive loads, and power flows from bus j to other buses, written as

$$P_j^e = D_j \omega_j + \sum_{l \in \mathcal{L}(j)} d_l + \sum_{k: j \to k} P_{jk} - \sum_{i: i \to j} P_{ij}. \tag{2}$$

Here P_{ij} is the deviation of branch flow from bus i to bus j from its nominal value. Our goal is to control the frequency-insensitive loads d_l in response to disturbances P_j^m in generation(mechanic) power. The swing equation can thus be rewritten as

$$\dot{\omega}_j = -\frac{1}{M_j} \left(\sum_{l \in \mathcal{L}(j)} d_l + D_j \omega_j - P_j^m + P_j^{\text{out}} - P_j^{\text{in}} \right), \tag{3}$$

where $P_j^{\text{out}} := \sum_{k: j \to k} P_{jk}$ and $P_j^{\text{in}} := \sum_{i: i \to j} P_{ij}$ are total branch power flows out and into bus j, respectively.

[1] These assumptions are similar to the standard DC approximation except that we do not assume the phase angle difference is small across each link.

We assume that the branch flows P_{ij} follow the dynamics

$$\dot{P}_{ij} = B_{ij} \, \omega^0 \left(\omega_i - \omega_j \right), \tag{4}$$

where ω^0 is the common nominal frequency on which the per-unit convention is based, and

$$B_{ij} := \frac{|V_i||V_j|}{x_{ij}} \cos\left(\theta_i^0 - \theta_j^0\right) \tag{5}$$

is a constant related to nominal voltages of buses and line reactance. The dynamic model (4)–(5) is motivated in the following way. Let θ_j denote the phase angle deviations of the bus voltages, i.e., the voltage phasors are $V_j := |V_j| e^{j(\theta_j^0 + \theta_j)}$ with the nominal phase angles θ_j^0. Consider the deviations in branch flows P_{ij} when the deviations are small [1], [7, Chapter 11]:

$$P_{ij} = B_{ij} (\theta_i - \theta_j), \tag{6}$$

and $\dot{\theta}_j = \omega^0 \omega_j$, then we have (4). Note that, while the model (6) assumes that the differences $\theta_i - \theta_j$ of the deviations are small, it does not assume the differences $\theta_i^0 - \theta_j^0$ of their nominal values are small.

In summary, the dynamic model of the transmission network is specified by (3)–(5). In steady state, the mechanic power deviations P_j^m are equal to the electric power deviations P_j^e, and $\omega_i = \omega_j$ for all the buses i and j, so $\dot{\omega}_j = 0$ and $\dot{P}_{ij} = 0$.

III. Optimal load control problem

Let a variable without a subscript denote a vector with appropriate components, e.g., $d := (d_l, l \in \mathcal{L}(j), j \in \mathcal{V})$, $\omega := (\omega_j, j \in \mathcal{V})$, $P := (P_{ij}, (i,j) \in \mathcal{E})$.

Suppose a step change $P^m = (P_1^m, \dots, P_N^m)$ in generation is injected to the N buses. How should the frequency-insensitive loads $d = (d_l, l \in \mathcal{L}(j), j \in \mathcal{V})$ in the network be reduced (or increased) in real-time in a way that (i) balances the generation shortfall (or surplus), (ii) resynchronizes the bus frequencies, and (iii) minimizes a measure of aggregate disutility of participation in such a load control? We formalize these questions as an *optimal load control (OLC)* problem.

The disturbance P^m in generation causes a nonzero frequency deviation ω_j at bus j. This frequency deviation incurs a cost to frequency-sensitive loads and suppose this cost is $\frac{1}{2D_j} \hat{d}_j^2$ in total at bus j. Suppose the frequency-insensitive load $l \in \mathcal{L}(j)$ is to be changed by an amount d_l which incurs a cost (disutility) of $c_l(d_l)$. We assume $-\infty < \underline{d}_l \le d_l \le \overline{d}_l < \infty$. Our goal is to minimize the total cost over $(d, \hat{d})$ while balancing generation and load across the network, written as *OLC*

$$\min_{\underline{d} \le d \le \overline{d}, \hat{d}} \sum_{j \in \mathcal{V}} \left(\sum_{l \in \mathcal{L}(j)} c_l(d_l) + \frac{1}{2D_j} \hat{d}_j^2 \right) \tag{7}$$

$$\text{subject to} \sum_{j \in \mathcal{V}} \left(\sum_{l \in \mathcal{L}(j)} d_l + \hat{d}_j \right) = \sum_{j \in \mathcal{V}} P_j^m. \tag{8}$$

We make the following assumption throughout the paper:

Assumption 1. The *OLC* is feasible, and the cost functions c_l are strictly convex and twice continuously differentiable on $\left[\underline{d}_l, \overline{d}_l\right]$.

IV. LOAD CONTROL AND SWING DYNAMICS AS PRIMAL-DUAL SOLUTION

In this section, we interpret how a frequency-based load control and the network dynamics serve as a distributed primal-dual algorithm to solve *OLC*. Due to space limitations, we put the proofs of theorems in our technical report [8].

A. Key results

The objective function of the dual problem of *OLC* is:

$$\sum_{j\in\mathcal{V}} \Phi_j(\nu) := \sum_{j\in\mathcal{V}} \min_{\underline{d}_l \le d_l \le \overline{d}_l, \hat{d}_j} \left(\sum_{l\in\mathcal{L}(j)} (c_l(d_l) - \nu d_l) + \left(\frac{1}{2D_j}\hat{d}_j^2 - \nu \hat{d}_j\right) + \nu P_j^m \right), \tag{9}$$

where Φ_j can be written as

$$\Phi_j(\nu) = \sum_{l\in\mathcal{L}(j)} \left(c_l(d_l(\nu)) - \nu d_l(\nu)\right) - \frac{D_j \nu^2}{2} + \nu P_j^m \tag{10}$$

with

$$d_l(\nu) := \left[c_l'^{-1}(\nu)\right]_{\underline{d}_l}^{\overline{d}_l}. \tag{11}$$

This objective function has a scalar variable ν and is not separable across buses $j = 1, \dots, N$. Its direct solution hence requires coordination across the buses. We propose a following *distributed* version of the dual problem where each bus j optimizes over its own variable ν_j, one of the multiple copies of ν that are constrained to be equal at optimality.

DOLC

$$\max_{\nu_j} \ \Phi(\nu) := \sum_{j\in\mathcal{V}} \Phi_j(\nu_j) \tag{12}$$

$$\text{subject to} \quad \nu_i = \nu_j \qquad \text{for all } (i,j) \in \mathcal{E}. \tag{13}$$

Theorem 1. The following statements hold.

1) *DOLC* has a unique optimal solution ν^* with $\nu_i^* = \nu_j^* = \nu^*$ for all $i, j \in \mathcal{V}$.[2]
2) *OLC* has a unique optimal solution $(d^*, \hat{d}^*)$ where $d_l^* = d_l^*(\nu^*)$ is given by (11) for all $l \in \mathcal{L}(j)$ and $j \in \mathcal{V}$, and $\hat{d}_j^* = D_j \nu^*$ for all $j \in \mathcal{V}$.
3) There is no duality gap.

We put the proof of Theorem 1 in [8, Section VII-A].

Instead of solving *OLC* directly, Theorem 1 suggests solving its dual *DOLC* and recovering the unique optimal solution $(d^*, \hat{d}^*)$ of the primal problem *OLC* from the unique dual optimal ν^*. To derive a distributed solution for *DOLC*, consider its Lagrangian

$$L(\nu, \pi) := \sum_{j\in\mathcal{V}} \Phi_j(\nu_j) - \sum_{(i,j)\in\mathcal{E}} \pi_{ij}(\nu_i - \nu_j), \tag{14}$$

[2] We abuse notation and use ν^* to denote both the vector and the common value of its components.

where ν is the (vector) variable for *DOLC* and π is the associated dual variable for the dual of *DOLC*. Hence π_{ij}, for all $(i,j) \in \mathcal{E}$, measure the cost of not synchronizing the variables ν_i and ν_j across buses i and j. A primal-dual algorithm for *DOLC* takes the form (using (10)–(11))

$$\begin{aligned} \dot{\nu}_j &= \gamma_j \frac{\partial L}{\partial \nu_j}(\nu, \pi) \\ &= -\gamma_j \left(\sum_{l\in\mathcal{L}(j)} d_l(\nu_j) + D_j \nu_j - P_j^m + \pi_j^{\text{out}} - \pi_j^{\text{in}} \right), \end{aligned} \tag{15}$$

$$\dot{\pi}_{ij} = -\xi_{ij} \frac{\partial L}{\partial \pi_{ij}}(\nu, \pi) \ = \ \xi_{ij}(\nu_i - \nu_j), \tag{16}$$

where $\gamma_j > 0$, $\xi_{ij} > 0$ are stepsizes and $\pi_j^{\text{out}} := \sum_{k:j\to k} \pi_{jk}$, $\pi_j^{\text{in}} := \sum_{i:i\to j} \pi_{ij}$.

It is then remarkable that (15)–(16) become identical to (3)–(4), if we identify ν with frequency deviations and π with branch flows, i.e.,

$$\nu_j = \omega_j, \ \ \pi_{ij} = P_{ij}, \tag{17}$$

and the stepsizes γ_i and ξ_{ij} with the system parameters

$$\gamma_j = M_j^{-1}, \ \ \xi_{ij} = B_{ij} w^0. \tag{18}$$

For convenience, we collect the system dynamics and load control:

$$\dot{\omega}_j = -\frac{1}{M_j} \left(\sum_{l\in\mathcal{L}(j)} d_l + \hat{d}_j - P_j^m + P_j^{\text{out}} - P_j^{\text{in}} \right) \tag{19}$$

$$\dot{P}_{ij} = B_{ij}\, \omega^0 \,(\omega_i - \omega_j) \tag{20}$$

$$\hat{d}_j(\omega_j) = D_j \omega_j \tag{21}$$

$$d_l(\omega_j) = \left[c_l'^{-1}(\omega_j)\right]_{\underline{d}_l}^{\overline{d}_l} \qquad \text{for all } l \in \mathcal{L}(j), \tag{22}$$

where $P_j^{\text{out}} = \sum_{k:j\to k} P_{jk}$ and $P_j^{\text{in}} = \sum_{i:i\to j} P_{ij}$ are total branch power flows out and into bus j, ω^0 is the common nominal frequency, and B_{ij} are given by (5). The dynamics (19)–(21) are automatically carried out by the power system while the local control (22) need to be implemented at each frequency-insensitive load.

We make the following assumption:

Assumption 2. For all $j \in \mathcal{V}$ and all $l \in \mathcal{L}(j)$, there exists some $\alpha_l > 0$ so that $c_l''(d_l) \ge 1/\alpha_l$ for $d_l \in \left[\underline{d}_l, \overline{d}_l\right]$. Moreover, $d_l'(\cdot) = \left((c_l')^{-1}\right)'(\cdot)$ is Lipschitz on $\left(c_l'\left(\underline{d}_l\right), c_l'\left(\overline{d}_l\right)\right)$.

Assumption 2 is satisfied for disutility functions that are commonly used for demand response, e.g., quadratic function.

Let $(d(t), \hat{d}(t), \omega(t), P(t))$ denote a trajectory of frequency-insensitive loads, frequency-sensitive loads, frequency deviations and power flows over time t, generated by the swing dynamics and the load control (19)–(22).

Theorem 2. Suppose Assumption 2 holds. Every trajectory $(d(t), \hat{d}(t), \omega(t), P(t))$ converges to a limit $(d^*, \hat{d}^*, \omega^*, P^*)$ as $t \to \infty$ such that

1) $(d^*, \hat{d}^*)$ is the unique vector of optimal load control for *OLC*;
2) ω^* is the unique vector of optimal frequency deviations for *DOLC*;
3) P^* is a vector of optimal branch flows for the dual of *DOLC*.

We put the proof of Theorem 2 in [8, Section IV], and give the sketch of proof as follows.

1) The set of optimal solutions (ω^*, P^*) of *DOLC* and its dual, the set of saddle points of the Lagrangian of *DOLC*, and the set of equilibrium points of (19)–(22) are all the same, denoted by Z^*.
2) Let $v := (\omega, P)$. Following [9], we consider the candidate Lyapunov function

$$U(v) = \frac{1}{2}\,(v - v^*)^T \begin{bmatrix} \Gamma^{-1} & 0 \\ 0 & \Xi^{-1} \end{bmatrix} (v - v^*)\,, \quad (23)$$

where $v^* = (\omega^*, P^*)$ is any point in Z^*, $\Gamma = \mathrm{diag}(M_j^{-1})$ and $\Xi = \omega^0 \mathrm{diag}(B_{ij})$. Using such a Lyapunov function U, we prove that every solution $(\omega(t), P(t))$ of (19)–(20) approaches a nonempty, compact subset Z^+ of $\mathcal{Z}^*$ as $t \to \infty$.
3) If the network graph $\mathcal{G}$ is a tree, $Z^+ = Z^*$ has a unique equilibrium point (ω^*, P^*), and every system trajectory $(d(t), \hat{d}(t), \omega(t), P(t))$ converges to the unique system optimal $(d^*, \hat{d}^*, \omega^*, P^*)$, i.e., it satisfies the three conclusions in Theorem 2.
4) If $\mathcal{G}$ is a mesh network, we consider $h(t) := CP(t)$, where C is the $N \times |\mathcal{E}|$ incidence matrix of $\mathcal{G}$, i.e., $C_{il} = 1$ if node i is the source of a directed link $l = (i,j)$, and $C_{il} = -1$ if node i is the sink of a directed link $l = (j,i)$. Then (19)–(20) becomes

$$\dot{\omega} = \Gamma \left(\left[\frac{\partial \Phi}{\partial \omega}\,(\omega(t)) \right]^T - h(t) \right), \quad (24)$$

$$\dot{h} = C\,\Xi\,C^T \omega(t). \quad (25)$$

We prove that (24)–(25) has a unique equilibrium (ω^*, h^*), which is not only globally asymptotically stable, but also, under Assumption 2, exponentially stable. Then, for all $(i,j) \in \mathcal{E}$, $P_{ij}(t)$ satisfies the Cauchy condition, and thus converges to a P_{ij}^* depending on the initial condition. It follows that $(d(t), \hat{d}(t), \omega(t), P(t))$ converges to a system optimal $(d^*, \hat{d}^*, \omega^*, P^*)$, i.e., it satisfies the three conclusions in Theorem 2.

B. Implications

Our results have several important implications:

1) *Frequency-based load control:* The frequency-insensitive loads can be controlled using their individual marginal cost functions according to (22), based only on frequency deviations $\omega_j(t)$ (from their nominal value) that are measured at their local buses.
2) *Complete decentralization.* Our result implies that the local frequency deviation $\omega_j(t)$ at each bus turns out to convey exactly the right information about the global power imbalance for the loads themselves to make optimal decisions based on their own marginal cost functions. This allows a completely decentralized solution without the need for explicit communication among the buses.
3) *Reverse engineering of swing dynamics.* The frequency-based load control (22) coupled with the dynamics (19)–(21) of swing equations and branch power flows serve as a distributed primal-dual algorithm to solve *OLC* and its dual *DOLC*.
4) *Frequency and branch flows.* In the context of optimal load control, the frequency deviations $\omega_j(t)$ emerge as the Lagrange multipliers of *OLC* that measure the cost of power imbalance, whereas the branch flow deviations $P_{ij}(t)$ emerge as the Lagrange multipliers of *DOLC* that measures the cost of frequency asynchronism.
5) *Uniqueness of solution.* Theorem 1 implies that the optimal frequency ω^* is unique and hence the optimal load control $(d^*, \hat{d}^*)$ is unique. As we showed, the optimal branch flows P^* are unique if and only if the network is a tree. Theorem 2 says nonetheless, that, even for mesh networks, any trajectory generated by the load control and swing dynamics indeed converges to an optimal point, with the optimal value of P^* dependent on the initial condition.
6) *Optimal frequency.* The structure of *DOLC* implies that the frequencies at all the buses are synchronized at optimality. Moreover, the common frequency *deviation* ω^* at optimality is in general nonzero. This fact implies that while frequency-based load control and the swing dynamics can resynchronize bus frequencies to a unique common value after a disturbance in generation, the new frequency may be different from the nominal value. Other mechanisms, such as automatic generation control, will be needed to drive the new operating frequency to its nominal value.

Of course, many of these insights are well known; our results merely provide a fresh and unified interpretation within an optimization framework for load control.

V. Case Studies

To test the performance of the proposed *OLC* mechanism, we run simulation on a 16-generator transmission network test bed built with MATLAB.

A. Transmission network model for simulation

We consider a simplified version of the 16-generator, 68-bus test system of the New England/ New York interconnection given in [11]. We simplify the network in [11] by grouping a generator bus with its nearby load buses to form a bus with both generation and loads. Thereby, we get a 16-generator, 16-bus network, as shown in Figure 1.

The values of generator and transmission line parameters and the values of parameters in *OLC* are shown in [8, Tables I and II]. We do not show them here due to space limitations.

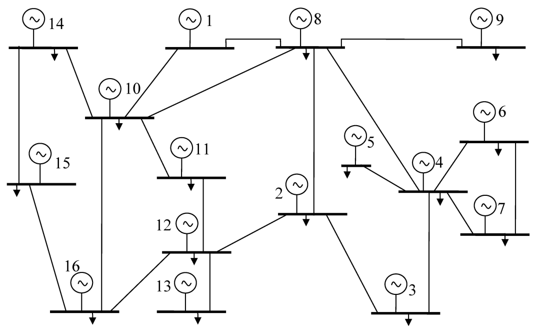

Fig. 1. The 16-generator transmission network model used in simulation. The arrows stand for controllable loads.

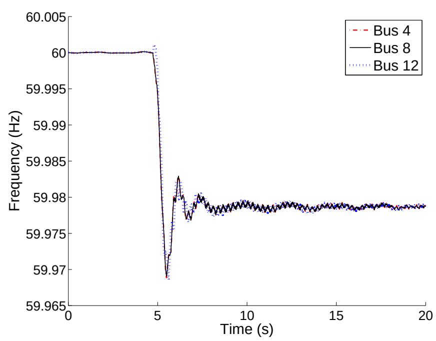

Fig. 2. Frequencies at buses 4, 8, 12, with loads performing *OLC*.

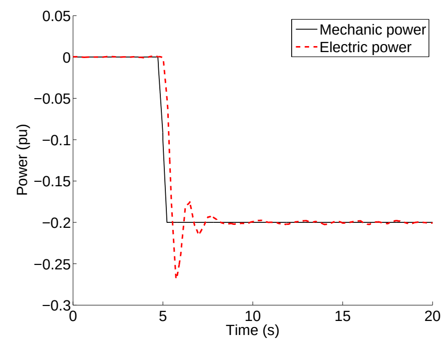

Fig. 3. Total change of mechanic power and electric power, with loads performing *OLC*.

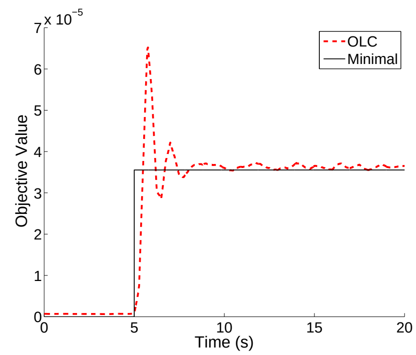

Fig. 4. Objective value of *OLC*.

In the network, every controllable load l has a cost function $c_l(d_l) = d_l^2/(2\alpha_l)$ on $d_l \in \left[\underline{d}_l, \overline{d}_l\right]$, where $\underline{d}_l < 0$ and $\overline{d}_l > 0$ are randomly generated subject to the bounds on total change in controllable loads given by [8, Table I], and $\alpha_l > 0$ is a random number. Here, we pick α_l uniformly distributed on $(0.2, 0.5)$.

In the model used for simulation, we relax some of the assumptions we made in previous sections. We consider non-zero line resistance and do not assume small differences between phase angle deviations. Moreover, at buses 1–12, the damping constant $D_j = 0$, and at bus 1, there are no controllable loads. In practice, the frequency measurement and load control cannot be performed continuously in time. Therefore, in simulation, the loads measure the frequency and control their power every 250 ms. Moreover, the frequency measurements have Gaussian errors with a standard deviation of 3×10^{-5} pu (1.8 mHz) [12].

B. Performance of OLC

At time $t = 5$ s, a step change of mechanic power occurs at buses 4, 8, 12. The controllable loads perform *OLC*. Figures 2–4 respectively show the frequencies at buses 4, 8, 12, the total change in mechanic power and electric power, and the objective value of *OLC*. We see that that frequencies at three buses are driven to the same value near 60 Hz, the total change in electric power balances the total change in mechanic power, and the objective value of *OLC* goes to the minimum, all in less than 5 seconds after the step change of mechanic power. There are oscillations around the steady states, which may be caused by discrete-time load control or frequency measurement errors. The oscillations are relatively small compared to the steady state values. We see that the *OLC* has satisfactory performance in resynchronizing bus frequencies, matching load with generation and minimizing the aggregate disutility.

C. Incorporating OLC with AGC

AGC has been widely used in the regulation of transmission network. Hence, we compare the performance of *OLC* with AGC, and look into the effect of their incorporation.

The model of bus j equipped with AGC is shown in Figure 5. In AGC, the generator controller computes area control error (ACE), which is a weighted sum of frequency deviation ω_j

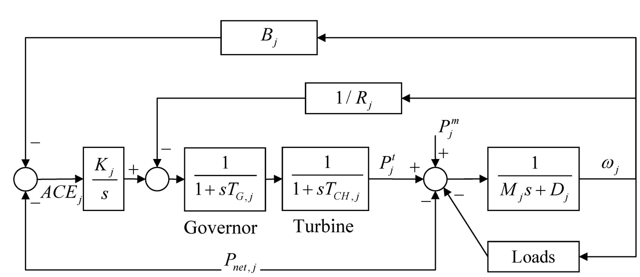

Fig. 5. Model of a bus equipped with AGC.

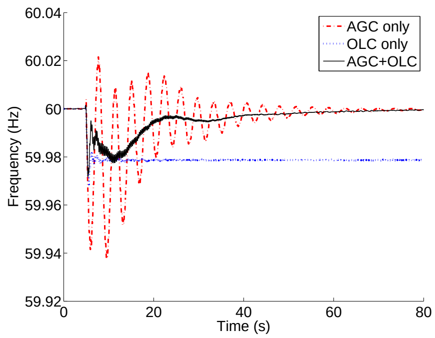

Fig. 6. Frequency at bus 12, with AGC only, with *OLC* only, or with both of them.

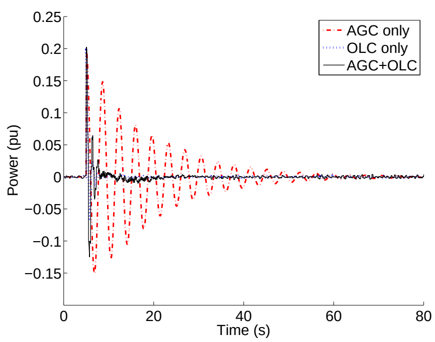

Fig. 7. Total mismatch between electric power and mechanic power, with AGC only, with *OLC* only, or with both of them.

and the unscheduled net power flow out of the area $P_{net,j} = \sum_{k:j\rightarrow k} P_{jk} - \sum_{i:i\rightarrow j} P_{ij}$. The setpoint of the governor is adjusted according to the integral of ACE, and the change of mechanic power output of the turbine, P_j^t, is controlled by the governor. To improve stability, the governor also takes negative feedback of ω_j by the gain $1/R_j$. The governor and the turbine respectively have a time constant $T_{G,j}$ and $T_{CH,j}$. For all the generators in the simulation, we take $R_j = 0.1$ pu, $K_j = 0.05$, and $B_j = 1/R_j + D_j$ [7]. Moreover, for all the generators, $T_{G,j} = 0.04$ s and $T_{CH,j} = 5$ s.

At time $t = 5$ s, a step change of mechanic power occurs at buses 4, 8, 12. Figures 6–7 respectively show the frequency at bus 12, and the total mismatch between electric power and mechanic power, in the case of using only AGC, using only *OLC* and incorporating both of them. With AGC only, the frequency is driven to 60 Hz, and electric power and mechanic power are balanced in about 1 minute. However, within the first minute, there are large overshootings and oscillations in both frequency and electric-mechanic power mismatch. With *OLC* only, electric power and mechanic power are balanced quickly, and the frequency is quickly driven to a steady state value which is not 60 Hz. When *OLC* is implemented together with AGC, the frequency can be driven to 60 Hz and electric power is balanced with mechanic power. Moreover, compared to the case of using AGC only, the settling time is decreased, and the overshootings and oscillations are significantly alleviated. The result shows that adding *OLC* can improve the transient performance of AGC.

VI. Conclusion

We proposed an *optimal load control (OLC)* problem in power transmission networks. The objective of *OLC* is to minimize the aggregate disutility of participation in load control, subject to the balance between total generation and load throughout the network. Then, we developed an equivalent problem of *OLC* through taking its dual problem, and designed a distributed primal-dual algorithm to solve that equivalent problem. The algorithm is composed by both the dynamics of the power network and a frequency-based load control mechanism. In the mechanism, loads are controlled as the inversed marginal disutility function of locally measured frequency. We proved that the trajectory produced by the algorithm convergences to the optimum of *OLC*. Simulation on a transmission network test bed showed that the proposed mechanism can resynchronize bus frequencies, balance load with generation and achieve the optimum of *OLC* quickly after a disturbance in generation. Simulation also showed that adding *OLC* can improve the transient performance of AGC.

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